COMPLEMENTARITY AND PARACONSISTENCY

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Abstract

Bohr's Principle of Complementarity is controversial and there has been much dispute over its precise meaning. Here, without trying to provide a detailed exegesis of Bohr's ideas, we take a very plausible interpretation of what may be understood by a theory which encompasses complementarity in a definite sense, which we term $\mathcal{C}$-theories. The underlying logic of such theories is a kind of logic which has been termed 'paraclassical', obtained from classical logic by a suitable modification of the notion of deduction. Roughly speaking, $\mathcal{C}$-theories are non-trivial theories which may have 'physically' incompatible theorems (and, in particular, contradictory theorems). So, their underlying logic is a kind of paraconsistent logic.

Keywords: Complementarity, Paraconsistency, Paraclassical Logic.

1 Introduction

"Ceci met en évidence l'apparence irrationnelle de la complémentarité qui ne se rationalise que par des schèmes logiques nouveaux."

P. Fèvrier (1951)

The concept of 'complementarity' was introduced in quantum mechanics by Niels Bohr in his famous 'Como Lecture', in 1927 (Bohr 1927). The consequences of his ideas were fundamental for the development of the Copenhagen interpretation of quantum mechanics and constitutes, as is largely recognized in the literature, as one of the most fundamental contributions to the development of quantum theory (see Beller 1992; Jammer 1966, 1974).

Notwithstanding their importance, Bohr's ideas on complementarity are controversial. In reality, it seems that there is no general agreement on the precise meaning of his *Principle of Complementarity* (see for instance Beller 1992, p. 148); Bohr's own words, by posing that "I think that it would be reasonable to say that no man who is called a philosopher really understands what is meant by complementary descriptions" (quoted from Cushing 1994, p. 32), might suggest the difficulties involved in any attempt to search for a 'rationale' for his Principle. Anyhow, this remark invites us to look also at the logico-mathematical grounds, mainly in connection with the paraconsistent program (see da Costa and Marconi 1987; da Costa and Bueno 2001).

So, although it has also been claimed that Bohr apparently understood the Principle of Complementarity from an epistemological point of view only (cf. Jammer 1974, pp. 70 and 89), we think that it is pertinent to ask for the logical structure of a theory which encompasses such a principle in its bases. Then, taking into account that the intuitive idea of complementarity resembles that of contradiction (see below), the underlying logical structure of such a theory should be made explicit.

As a historical remark, we recall that some authors like C. von Weizsäcker, M. Strauss and P. Fèvrier already tried to elucidate Bohr's principle from a logical point of view (cf. Fèvrier 1951; Jammer 1974, pp. 376ff; Strauss 1973, 1975); Jammer mentions Bohr's negative answer to von Weizsäcker's attempt of interpreting Bohr's principle and observes that this should be taken as a warning for analyzing the subject (ibid. p. 90). He also mentions that Strauss' intention was to develop a logic in which two propositions, say α and β (which should stand for complementary propositions) may be both accepted as true, but not their conjunction $\alpha \wedge \beta$ (ibid., p. 335); R. Carnap suggests that Strauss' logic were 'inadvisable' (Carnap 1995, p. 289).

The introduction of some non-classical logical systems developed more recently may enrich the discussion, and this is what we are doing now. But let us first recall that apparently 'complementary descriptions' are more concerned with 'exclusive descriptions' than with the impossibility of 'simultaneous measurement', as implicitly suggested in some standard books when they 'define' complementarity (see for instance Omnès 1995).

We shall proceed as follows. Without discussing von Weizsäcker's or Strauss' works (only Février's ideas will be mentioned in brief below in order to motivate the paper), we introduce the concept of a theory which admits a *Complementarity Interpretation* (to use Jammer's words –see below). Then we suggest that under a plausible interpretation of what is to be understood by complementarity, the underlying logic of such a theory is a *paraclassical* logic (first proposed in da Costa and Vernengo 1999). Below we shall sketch the main features of this logic as applied to our purposes.

En passant, let us mention that one thing is to provide an exegesis of Bohr's ideas; another is to pay attention to the underlying logical structure of a theory which encompasses complementarity in some sense. In this paper, although we regard the first topic as very important, we are fundamentally concerned with the second, even if we do not provide all the technical details, which will be postponed to future technical works. So, this paper can be regarded as an adjunct to the speculations on this second point. Concerning the first point, see Beller (1992) for a detailed attempt to 'decipher' Bohr's principle "by uncovering and describing the underlying network of implicit dialogues in the Como lecture".

Finally, let us say that our paper might be also viewed as an attempt to investigate a line of research which was envisaged, but not developed, by P. Février; in short, she attributed a third value (*impossible*) to the conjunction of complementary propositions (*propositions incomposables*) so that her logic resembles Łukasiewicz' three valued logic (Jammer's book provides a general view on these logics; see Jammer 1974, pp. 341ff). Notwithstanding, Février recognized that we could also consider that the conjunction of complementary propositions cannot be performed: "la conjonction 'et' ne peut leur être appliquée" (Fèvrier 1951, p. 33), but she did not consider such a possibility due to "raisons de technique mathématique" (ibid.). In this paper we articulate a possible way to supersede these 'difficulties', motivated by the paraconsistent program, which at that time had not yet been developed. Our approach runs in the direction of not avoiding that the conjunction of complementary sentences can be performed but, roughly speaking, that such a conjunction cannot be derived as a theorem of the theory.

In our opinion, Bohr's view provides the grounds for defining a very general class of theories, which may be regarded as theories which incorporate axioms that may entail propositions like γ and $\neg\gamma$ (the negation of γ), but such that the theory is not trivial in the sense that this fact does not imply that all the formulas of its language are theorems, as we shall see below. In other words, the theories we shall characterize below are such that from γ and $\neg\gamma$ we cannot deduce $\gamma \wedge \neg\gamma$, that is, a contradiction.

We should still remark that this kind of investigation has not only historical reasons, as one should infer from the fact that nowadays the concept of complementarity seems to be no more popular among physicists. Really, the investigation of the logical foundations of science has a value by itself, and the resulting systems (when they arise, as in the present case), built as sometimes motivated by not so clear intuitions, not only may provide them a sense according to acceptable paterns of rigour, but they also may be useful in other situations as well, which may provide other insights and further developments. Furthermore, our work shows that by taking the concept of complementarity as we have considered it (see the next section), there is a sense in saying that the founders of quantum theory, in particular Bohr, may be referred to as 'inconsistent', as probably are all those who are developing very creative efforts, but for sure their feelings were not trivial in the sense defined below. Maybe we could say, taking the due care: they are paraconsistent.

2 A way of understanding complementarity

In order to explain the sense according to which we shall consider the term 'complementarity' in this paper, let us look at how this concept was analyzed by some authors.

Of course, a few isolated quotations cannot provide evidence for the understanding of concepts, especially regarding the present case, but perhaps we could reinforce our point by showing that complementarity stands more for 'incompatibility' in some sense (the 'sense' being explained in the next sections) than for impossibility of 'simultaneously measuring', an expression which could resemble the use of some kind of temporal logic.

Anyway, it should be remarked that we may also find Bohr speaking about complementary concepts which cannot be used *at the same time* (as we can see in several papers in Bohr 1985), but these situations according to him demand isolated analyses, and perhaps it is not possible to provide a general description which allows us to deal with all of these cases: according to Bohr, "One must be very careful, therefore, in analyzing which concepts actually underly limitations" (ibid., p. 369).

Pauli, for instance, has claimed that, "[If] the use of a classical concept excludes of *another*, we call both concepts (...) *complementary* (to each other), following Bohr" (Pauli 1980, p. 7, quoted in Cushing 1994, p. 33). Cushing has also stressed that, "[W]hatever historical route, Bohr did arrive at a doctrine of mutually exclusive, incompatible, but necessary classical pictures in which any given application emphasizing one class of concepts *must* exclude the other" (ibid., pp. 34-5).

This idea that complementary propositions 'exclude' each other (incompatibility) is reinforced by Bohr himself in several passages:

> The existence of different aspects of the description of a physical system, seemingly incompatible but both needed for a complete description of the system. In particular, the wave-particle duality. (quoted from French and Kennedy 1985, p. 370)
>
> The phenomenon by which, in the atomic domain, objects exhibit the properties of both particle and waves, which in classical, macroscopic physics are mutually exclusive categories. (ibid., pp. 371-2)
>
> The very nature of the quantum theory thus forces us to regard the space-time co-ordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description, symbolizing the idealization of observation and definition respectively. (Bohr 1927, p. 566)

Several other passages from Bohr could be quoted from Scheibe's book (1973), for instance, the following:

> The apparently incompatible sorts of information about the behavior of the object under examination which we get by different experimental arrangements can clearly not be brought into connection with each other in the usual way, but may, as equally essential for an exhaustive account of all experience, be regarded as 'complementary' to each other. (Bohr 1937, p. 291; Scheibe 1973, p. 31)

Scheibe also says that

> ...which is here said to be 'complementary', is also said to be 'apparently incompatible', the reference can scarcely be to those classical concepts, quantities or aspects whose *combination* was previously asserted to be characteristic of the classical theories. For 'apparently incompatible' surely means incompatible on classical considerations alone. (Scheibe 1973, p. 31)

The following quotation is also relevant for the point we are trying to stress here: the characteristic of 'exclusion' of complementarity. Bohr says:

> Information regarding the behaviour of an atomic object obtained under definite experimental conditions may, however, according to a terminology often used in atomic physics, be adequately characterized as *complementary* to any information about the same object obtained by some other experimental arrangement excluding the fulfillment of the first conditions. Although such kinds of information *cannot be combined into a single picture* by means of ordinary concepts, they represent indeed equally essential aspects of any knowledge of the object in question which can be obtained in this domain. (Bohr 1938, p. 26, quoted from Scheibe 1973, p. 31, second italic ours).

In other words, it seems perfectly reasonable to regard complementary aspects as *incompatible*, in the sense that their *combination* into a single description may lead to difficulties. In this sense, the quantum world is rather distinct from the 'classical' world.

It should be remarked that in the 'classical world', which at first glance can be described by using standard logic and mathematics, if α and β are both theses or theorems of a theory (founded on classical logic), then $\alpha \wedge \beta$ is also a thesis of that theory. This is what we intuitively mean when we say that on the grounds of classical logic, a true proposition cannot 'exclude' another true proposition.

In classical logic, if from some group Δ_1 of axioms of a theory T we deduce γ, and if from another group Δ_2 we deduce $\neg\gamma$, then $\gamma \wedge \neg\gamma$ is also deductible in T.

Normally, our group Δ of axioms of T is *finite*, so that we may talk of the conjunction of its sentences instead of Δ itself. Then, if α and β are respectively the conjunctions associated to Δ_1 and Δ_2, as above, we are looking for a theory T such that in T we may have $\alpha \vdash \gamma$ and $\beta \vdash \neg\gamma$, but in which $\gamma \wedge \neg\gamma$ is not a theorem of T.

Therefore, our goal is to describe a way to formally avoid that $\Delta_1 \cup \Delta_2$ (or $\alpha \wedge \beta$) entails a contradiction, since we do not intend to rule out 'complementary situations'. Notwithstanding, we emphasize that Bohr's ideas are not completely clear, as the following quotation shows:

> The term ‘complementarity’, which is already coming into use, may perhaps be more suited also to remind us of the fact that it is the combination of features which are united in the classical mode of description but appear separated in the quantum theory that ultimately allows us to consider the latter as a natural generalization of the classical physical theories. (Bohr 1929, p. 19)

Anyhow, the treatment of complementarity given below can cope with this more general view of this concept.

3 $\mathcal{C}$-theories

In order to provide a more adequate idea about the manner we consider complementary propositions, let us quote Max Jammer:

> Although it is not easy, as we see, to define Bohr’s notion of *complementarity*, the notion of *complementarity interpretation* seems to raise fewer definitory difficulties. The following definition of this notion suggests itself. A given theory T admits a complementarity interpretation if the following conditions are satisfied: (1) T contains (at least) two descriptions D_1 and D_2 of its substance-matter; (2) D_1 and D_2 refer to the same universe of discourse U (in Bohr’s case, microphysics); (3) neither D_1 nor D_2, if taken alone, accounts exhaustively for all phenomena of U; (4) D_1 and D_2 are mutually exclusive in the sense that their combination into a single description would lead to logical contradictions.
>
> That these conditions characterize a complementarity interpretation as understood by the Copenhagen school can easily be documented. According to Léon Rosenfeld, (...) one of the principal spokesmen of this school, complementarity is the answer to the following question: What are we to do when we are confronted with such situation, in which we have to use two concepts that are mutually exclusive, and yet both of them necessary for a complete description of the phenomena? “Complementarity denotes the logical relation, of quite a new type, between concepts which are mutually exclusive, and which therefore cannot be considered at the same time –that would lead to logical mistakes– but which nevertheless must both be used in order to give a complete description of the situation.” Or to quote Bohr himself concerning condition (4): “In quantum physics evidence about atomic objects by different experimental arrangements (...) appears contradictory when combination into a single picture is attempted.” (...) In fact, Bohr’s Como lecture with its emphasis on the mutual exclusive but simultaneous necessity of the causal (D_1) and the space-time description (D_2), that is, Bohr’s first pronouncement of his complementarity interpretation, forms an example which fully conforms with the preceding definition. Borh’s

> discovery of complementarity, it is often said, constitutes his greatest contribution to the philosophy of modern science. (Jammer 1974, pp. 104-5)

Jammer's quotation will be interpreted as follows. Firstly, we shall take for granted that both D_1 and D_2 are sentences formulated in the language of a theory T and that they refer to the same universe of discourse, so that D_1 and D_2 can be formulated in its language. So, items (1) and (2) will be considered only implicitly. Item (3) will be understood as entailing that *both* D_1 and D_2 are, from the point of view of T, *necessary* for the full comprehension of the relevant aspects of the objects of the domain; so, we shall take both D_1 and D_2 as 'true' sentences (in an adequate 'model' of T). Item (4) deserves further attention. Jammer says that 'mutually exclusive' means that the "combination of D_1 and D_2 into a single description would lead to logical contradictions", and this is reinforced by Rosenfeld's words that the concepts "cannot be considered at the same time", since this would entail a "logical mistake". Then, we will informally say that 'mutually exclusive', or complementary, are incompatible sentences or propositions whose conjunction lead to a contradiction (in a theory T based on classical logic).

So, following Jammer and Rosenfeld (according to the above quotation), we shall say that a theory T admits complementarity interpretation, or that T is a $\mathcal{C}$-theory, if T encompasses non equivalent true formulas α and β (which may stand for Jammer's D_1 and D_2 respectively) about its particular universe of discourse such that they are 'mutually exclusive' in the sense that their conjunction yields to a contradiction in T, according to classical logic.

The problem with the above characterization of complementary sentences is that if the underlying logic of T is classical logic or, say, intuicionistic logic, then T is contradictory or inconsistent. Apparently, it is precisely this what Rosenfeld claimed in the above quotation. Obviously, if we intend to maintain the idea of complementary propositions in the sense described above, we must change the underlying logic of T, in particular, the way we 'deduce' things. So, we shall modify the classical concept of deduction, obtaining a new kind of logic, called *paraclassical* logic (cf. da Costa and Vernengo 1999).

4 The underlying logic of $\mathcal{C}$-theories

As we have remarked, if a theory T that admits complementarity is based on classical logic or even on the most usual systems of logic, then the existence of mutually exclusive theorems as described in the previous section implies that T is trivial, that is, all formulas of the language of T are theorems of T. But there is the possibility of using a convenient type of logical system to found such a theory T; by this way, we shall be able to treat situations in which γ and $\neg\gamma$ are both theorems of T but $\gamma \wedge \neg\gamma$ is not. So, if conveniently introduced, such logic will allow us to deal, in T, with the desirable 'complementary propositions' without contradiction and triviality or, in Rosenfeld's words quoted above, without danger of a "logical mistake". In what follows we shall delineate the basic ideas of such a logic.

In da Costa and Vernengo 1999, a new way of dealing with nontrivial systems was proposed. The logic presented in that paper can also be useful in situations that encompass complementarity. This kind of logic is a paraconsistent logic (according to the characterization of such logics described in da Costa and Marconi 1987; da Costa and Bueno 2001) and it is very well suited for our purposes. Since this logic is still not well known, we shall recall here its main features and emphasize those aspects that are relevant for our purposes. After this we show how such logic can be used as the underlying logic of $\mathcal{C}$-theories, and in the last section we sketch a way to generalize the ideas presented. As in [12], we shall be restricted to the propositional level of the new logic P, but of course it is easy to extent P to a first-order or even to higher-order systems.

Let C be an axiomatized system of the classical propositional calculus. The concept of deduction of C is the standard one; we use the symbol $\vdash$ to represent deductions in C. Furthermore, the formulas of C are denoted by Greek lowercase letters, while Greek uppercase letters stand for sets of formulas. The symbols $\neg$, $\rightarrow$, $\wedge$, $\vee$ and $\leftrightarrow$ have their usual meanings, and standard conventions in the writing of formulas will be also assumed without further comments. All the syntactical concepts and details may be found in Mendelson 1987. In particular, we are interested in the following definitions: a theory T (a set of formulas closed under deduction) is **inconsistent** if it contains a theorem α whose negation $\neg\alpha$ is also a theorem of T; otherwise, T is **consistent**. If $\mathcal{F}$ denotes the set of all formulas of the language of C, then T is **trivial** if the set of its theorems coincides with $\mathcal{F}$; otherwise, T is **nontrivial**.

All syntactical concepts of P are similar to the corresponding concepts of C. The notion of deduction is introduced as follows:

Definition 4.1 *Let Γ be a set of formulas of P and let α be a formula (of the language of P). Then we say that α is a (syntactical) P-consequence of Γ, and write*

$$\Gamma \vdash_{\mathsf{P}} \alpha$$

if and only if

(P1) $\alpha \in \Gamma$, *or*

(P2) *There exists a consistent (according to classical logic) subset $\Delta \subseteq \Gamma$ such that $\Delta \vdash \alpha$ (in classical logic).*

We call $\vdash_{\mathsf{P}}$ the relation of P-**consequence**. It is immediate that, among others, the following results can be proved:

Theorem 4.1

1. *If α is a theorem of the classical propositional calculus C and if Γ is a set of formulas, then $\Gamma \vdash_{\mathsf{P}} \alpha$. In particular, $\vdash_{\mathsf{P}} \alpha$.*

2. *If Γ is consistent (according to* C*), then* $\Gamma \vdash \alpha$ *(in* C*) iff* $\Gamma \vdash_{\mathsf{P}} \alpha$ *(in* P*).*

3. *If* $\Gamma \vdash_{\mathsf{P}} \alpha$ *and if* $\Gamma \subseteq \Delta$*, then* $\Delta \vdash_{\mathsf{P}} \alpha$ *(The defined notion of* P*-consequence is monotonic.)*

4. *The notion of* P*-consequence* ($\vdash_{\mathsf{P}}$) *is recursive.*

5. *Since the theses of* P *are the theses of* C*,* P *is decidable.*

Definition 4.2 *A set of formulas* Γ *is* P-**trivial** *iff* $\Gamma \vdash_{\mathsf{P}} \alpha$ *for every formula* α*. Otherwise,* Γ *is* P-**nontrivial**.

Definition 4.3 *A set of formulas* Γ *is* P-**inconsistent** *if there exists a formula* α *such that* $\Gamma \vdash_{\mathsf{P}} \alpha$ *and* $\Gamma \vdash_{\mathsf{P}} \neg\alpha$*. Otherwise,* Γ *is* P-**consistent**.

Theorem 4.2

1. *If* α *is an atomic formula, then* $\Gamma = \{\alpha, \neg\alpha\}$ *is* P*-inconsistent, but* P*-nontrivial.*

2. *If the set of formulas* Γ *is* P*-trivial, then it is trivial (according to classical logic). If* Γ *is nontrivial, then it is* P*-nontrivial.*

3. *If* Γ *is* P*-inconsistent, then it is inconsistent according to classical logic. If* Γ *is consistent according to classical logic, then* Γ *is* P*-consistent.*

A semantical analysis of P, for instance a completeness theorem, can be obtained without difficulty, as indicated in da Costa and Vernengo 1999.

We remark that $\{\alpha \wedge \neg\alpha\}$ is trivial in classical logic, but not P-trivial. Notwithstanding, we are not suggesting that complementary propositions should be understood as pairs of contradictory sentences.

Definition 4.4 *A* $\mathcal{C}$-**theory** *is a set of formulas* T *closed under the relation of* P*-consequence* $\vdash_{\mathsf{P}}$*, that is,* $\alpha \in T$ *for whatever* α *such that* $T \vdash_{\mathsf{P}} \alpha$*. In other words,* T *is a theory whose underlying logic is* P*.*

Theorem 4.3 *There exist* $\mathcal{C}$*-theories that are inconsistent from the point of view of classical logic, though* P*-nontrivial.*

Proof: Immediate consequence of Theorem 4.2. $\dashv$

In the common applications, the existence of consistent sets of formulas are usually assumed only in an informal way, as an implicit postulate. Intuitively speaking, it makes reference to the fact that some 'classical' (that is, based on usual mathematics) theories and hypotheses scientists accept are thought of as not contradictory (as consistent) in principle.

Theorem 4.4 *Every consistent classical theory, that is, every consistent theory founded in classical logic (and set theory) is a particular case of* $\mathcal{C}$*-theories.*

Finally, we state a result (the theorem below), whose proof is an immediate consequence of the above definition of P-consequence, that links our logic with the characterization of 'complementary propositions' presented above. Before this, we make a definition:

Definition 4.5 *Let T be a $\mathcal{C}$-theory and let α and β be formulas of the language of T. We say that α and β are T-***complementary** *(or simply* **complementary***) if there exists a formula γ of the language of T such that:*

1. $T \vdash_{\mathsf{P}} \alpha$ *and* $T \vdash_{\mathsf{P}} \beta$
2. $\alpha \vdash_{\mathsf{P}} \gamma$ *and* $\beta \vdash_{\mathsf{P}} \neg\gamma$

It is immediate that contradictory propositions like α and $\neg\alpha$ are complementary in the above sense, but once more we remark that we are not arguing that this particular logical situation constitute a condensed account of all Bohr's ideas, as those involved in the quotation shown in the end of the section 2. The interesting case results from the following theorem.

Theorem 4.5 *If α and β are complementary theorems of a $\mathcal{C}$-theory T and $\alpha \vdash_{\mathsf{P}} \gamma$ and $\beta \vdash_{\mathsf{P}} \neg\gamma$, then in general $\gamma \wedge \neg\gamma$ is not a theorem of T.*

Proof: Immediate, as a consequence of Theorem 4.2. $\dashv$

This result is in fact interesting, since we may admit propositions (complementary propositions) so that one of them entails a proposition while the another one entails the negation of such a proposition, but we cannot deduce that their conjunction entails a contradiction. As an example of a situation involving $\mathcal{C}$-theories, suppose that our theory T is classical mechanics, which can be axiomatized by means of a set-theoretical predicate (see Suppes 2002, Chap. 7), and that to the axioms of T we add the following ones:

(Ax1) p is a particle

(Ax2) p is a wave

Since (Ax2) implies the negation of (Ax1) (and reciprocally), T may be viewed as an example of a $\mathcal{C}$-theory, for in T we can derive both, 'p is a particle' and 'p is not a particle', but we cannot infer 'p is a particle *and* p is not a particle', which of course has no sense in physics. So, it seems reasonable to assume that the underlying logic of T is the paraclassical logic P.

The basic characteristic of T as a $\mathcal{C}$-theory is that in making inferences, we suppose that some hypotheses we handle are consistent. In other words, $\mathcal{C}$-theories are closer to those theories scientists *actually* use in their day-to-day activity than theories encompassing the classical concept of deduction.

5 The paralogic associated to a logic $\mathcal{L}$.

The technique used in this paper to define the paraclassical logic associated with classical logic can be generalized to any logic $\mathcal{L}$ (including logics having no negation symbol, but we will not deal with this case here). More precisely, starting from a logic $\mathcal{L}$, we can define the $\mathsf{P}_{\mathcal{L}}$-logic associated to $\mathcal{L}$ (the 'paralogic' associated to $\mathcal{L}$) as follows.

Let $\mathcal{L}$ be a logic, which may be classical logic, intuicionistic logic, some paraconsistent logic or, in principle, any other logical system. The deduction symbol of $\mathcal{L}$ is $\vdash_{\mathcal{L}}$, and it is defined according to the standards of the particular logic being considered. We still suppose that the language of $\mathcal{L}$ has a symbol for negation, $\neg$.

Definition 5.1 *A* **theory** *based on $\mathcal{L}$ (an $\mathcal{L}$-***theory***) is a set of formulas Γ of the language of $\mathcal{L}$ which is closed under $\vdash_{\mathcal{L}}$. In other words, $\alpha \in \Gamma$ for every formula α such that $\Gamma \vdash_{\mathcal{L}} \alpha$.*

Definition 5.2 *An $\mathcal{L}$-theory Γ is $\mathcal{L}$-***inconsistent** *if there exists a formula α of the language of $\mathcal{L}$ such that $\Gamma \vdash_{\mathcal{L}} \alpha$ and $\Gamma \vdash_{\mathcal{L}} \neg\alpha$, where $\neg\alpha$ is the negation of α. Otherwise, Γ is $\mathcal{L}$-***consistent***.*

Definition 5.3 *A $\mathcal{L}$-theory Γ is $\mathcal{L}$-***trivial** *if $\Gamma \vdash_{\mathcal{L}} \alpha$ for any formula α of the language of $\mathcal{L}$. Otherwise, Γ is $\mathcal{L}$-***nontrivial***.*

Then, we define the $\mathsf{P}_{\mathcal{L}}$-logic associated with $\mathcal{L}$ whose language and syntactical concepts are those of $\mathcal{L}$ but by modifying the concept of deduction as follows: we say that α is a $\mathsf{P}_{\mathcal{L}}$-**syntactical consequence** of a set Γ of formulas, and write $\Gamma \vdash_{\mathsf{P}_{\mathcal{L}}} \alpha$ iff:

1. $\alpha \in \Gamma$, or
2. There exists $\Delta \subseteq \Gamma$ such that Δ is $\mathcal{L}$-nontrivial, and $\Delta \vdash_{\mathcal{L}} \alpha$.

For instance, we may consider the paraconsistent calculus $\mathcal{C}_1$ [11] as our logic $\mathcal{L}$. Then the paralogic associated with $\mathcal{C}_1$ is a kind of 'para-paraconsistent' logic.

It seems worthwhile to note the following in connection with the paraclassical treatment of theories. Sometimes, when one has a paraclassical theory T such that $T \vdash_{\mathsf{P}} \alpha$ and $T \vdash_{\mathsf{P}} \neg\alpha$, there exist *appropriate* propositions β and γ such that T can be replaced by a classical consistent theory T' in which $\beta \to \alpha$ and $\gamma \to \neg\alpha$ are theorems. If this happens, the logical difficulty is in principle eliminable and classical logic maintained.

6 More general complementary situations

As it is well known, Bohr tried to apply his principle of complementarity to other fields of knowledge (cf. Jammer 1974). More recently, Englert et al. (1994) have suggested that complementarity is not simply a consequence of the uncertainty relations, as advocated by those who believe that "two complementary variables, such as position and momentum, cannot simultaneously be measured to less than a fundamental limit of accuracy" (op. cit.), but that

> (...) uncertainty is not the only enforce of complementarity. We devised and analysed both real and thought experiments that bypass the uncertainty relation, in effect to 'trick' the quantum objects under study. Nevertheless, the results always reveal that nature safeguards itself against such intrusions —complementarity remains intact even when the uncertainty relation plays no role. We conclude that complementarity is deeper than has been appreciated: it is more general and more fundamental to quantum mechanics than is the uncertainty rule. (ibid.)

If Englert et al. are right, then it seems that paraclassical logic can be useful also to treat those theories which encompass complementarity in their sense.

Anyway, this kind of logic can be also modified to cope with more general kinds of incompatibility, say 'physical incompatibility', incorporating physical incompatible postulates, so as characteristics of the behaviour of human beings, etc., but we shall leave this topic for another work.

7 Acknowledgments

The authors would like to thank Prof. Osvaldo Pessoa Jr. and the anonymous referees for useful remarks and comments.

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