SEPARABLE NON-INDIVIDUALS

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Abstract

We argue that in the informal semantics of quantum theories, we consider (even implicitly) the existence of collections of *non-individual* objects of some kind. But this assumption seems to be in disagreement with the idea of separability, for separability apparently entails individuality. In this paper we show that by a suitable change in the logico-mathematical basis, we have the grounds for saying that there may exist *separable* objects which do not have individuality. So we conclude that a form of *realism of non-individuals* is compatible with the standard formalism of quantum mechanics.

Introduction

The Bohr-Einstein dialogue is usually referred to as the most important scientific philosophical debate of the XXth century. As it is well known, the central issue was about the 'completeness' of quantum mechanics, defended by Bohr and criticized by Einstein. In short, Einstein claimed that there are "elements of reality", which can be measured, but cannot be accounted within the formalism of quantum mechanics. The famous EPR (Einstein-Podolski-Rosen) *Gedankenexperiment* was elaborated just for trying to show the incompleteness of quantum mechanics. Bohr reacted to this idea, and was taken by many as the winner of the debate (for a good description on this interesting history, see Ghirardi 2005: Chap.7).

From the sixties, after Bell's theorem, which shows that in the quantum formalism we can derive certain inequalities which show that non-locality (see below) is a characteristic fact of quantum reality, the core of the problem left to be just an epistemological one, and gained the status of what Shimony has termed 'experimental metaphysics' (Ghiradi (op.cit.): 126; but see also pp. 226 and 243), for there could be an experimental test for the existence of non-local phenomena. Local realism, defended by Einstein, combines *locality*, that is, the hypothesis that nothing (no body, no information) can travel faster than light and *realism* (a name for a lot of conceptions) which here can be summed up by saying that an outside world exists independently of our will or of our consciousness.[1] The existence of non-local phenomena was one of the most basic traits of Bohr's account to quantum mechanics; in short, according to this view, separated quantum systems S_1 and S_2 which have interacted in the past, even if now very far from one another, are still regarded as forming just one physical system, so that the mathematical description of the whole system would be done by a wave function $\psi_{12} = \psi_1 + \psi_2$ which cannot be factorized into two functions, one for each particular system. So, the formalism is so that in measuring a certain observable for S_1, we ‘telepathically’ (Einstein’s term) change the situation of S_2. This causes, according to Einstein, violation of locality and, in admitting it, we are “denying independent real situations as such to things which are spatially separated from each other”. Both situations he regarded as “unacceptable” (Einstein ,1959: 85).

Several experiments testing Bell’s inequalities were executed from the seventies on, culminating with the most famous one, performed by Aspect and his group from 1983, and most of them were in agreement with quantum mechanics and against Einstein’s view. The credo that Bohr’s account was true suggested that Einstein's local-realism should be ruled out.

It should be remarked that Einstein’s philosophy was strongly grounded in both locality and separability. Really, as put clearly by Howard (Howard, 1985), after the EPR paper Einstein presented his own version of the incompleteness argument (it is today well known that Einstein did not agree with Podolski's final version of the paper), which is entailed by the conjunction of these two assumptions. By separability, we mean (following Howard) that spatially separated systems have associated with them independent real states of affairs (Howard, 2004). To Einstein, suggests Howard, realism "… is not a philosophical doctrine about the interpretation of scientific theories or the semantics of theoretical terms (…) [but it] is a physical postulate, one of the most interesting kind" (ibid.), which bases his conception of physical reality.

The link between realism and separability can be better viewed in the following passage by Einstein himself, according to whom physical reality entails

> “(…) that what we conceive as existing ('actual') should somehow be localized in time and space. That is, the real in one part of space, A, should (in theory) somehow ‘exist' independently of that which is thought as real in another part of space, B. If a physical system stretches over the parts A *and* B, then what is present in B should somehow have an existence independent of what is present in A. What is actually present in B should thus not depend upon the type of measurement carried out in the part of the space, A; it should also be independent of whether or not, after all, a measurement is made in A." (apud Howard, 2004)

That is, as put by Howard, "[r]ealism is thus the thesis of spatial separability, the claim that spatial separation is a sufficient condition for the individuation of physical systems, and its assumption is here made into almost a necessary condition for the possibility of an intelligible science of physics" (ibid.). To reinforce this point, let us recall that in a letter to Born, Einstein said that

> “… if one abandons the assumption that what exists in different parts of space has its own, independent, real existence, then I simply cannot see what it is that physics is meant to describe. For what is thought to be a ‘system' is, after all, just a convention, and I cannot see how one could divide the world objectively in such a way that one could make statements about parts of it". (cf. French and Krause , 2005: Chap. 4).

The criticism usually addressed to the principle of separability seems to be centered in the fact (for instance, given by Howard himself) that it acts as a principle of individuation for quantum systems. As *individuals*, quantum objects would not be able to enter in entangled states, for in this case they *should* encompass peculiar properties, contrary to the idea of entanglement. Hence, separability, acting as a sufficient condition for individuality, suggests that the violation of Bell inequalities would entail, first, that we must give up separability for quantum particles in entangled states and hence, secondly, that when in such states these particles cannot be regarded as individuals. In other words, Bell's Theorem implies *non-individuality* (French and Krause, loc.cit.).

We think that we need to eliminate separability from the quantum realm only if we regard quantum systems to be individuals according to the 'classical ontology', as we shall try to show in the last section. But, if we keep our formalism compatible with a metaphysics of non-individuals, that is, of entities (for the lack of a better term) devoid of identity, but so that can form collections of them with cardinal greater that one, then we can reinterpret separability in terms of the existence of such collections, and so admit that we do have *physical objects* of a kind, although non-individuals. After describing the main characteristics of 'classical ontology', we turn to a delineation of our metaphysics of non-individuals.

Classical and Quantum Ontologies

Following Mittelstaedt (2003) and Redhead and Teller (1991), we can describe the main features of 'classical' physical objects, so characterizing *classical ontology*, O(C). The objects of O(C) have (among others) the following characteristics:

(1) They are *continuants,* in the sense that an individual at one time can be identified as the same one that existed at an earlier time –that is, they have *genidentity*, or trans-temporal identity (at least during a certain interval of time).

(2) They are objects of predication, and may bear properties.

(3) They are *individualizable*, in the sense that there is `something' which confers them an identity.[2]

(4) They have identity;

(5) They can be ordered, counted, named, tagged, labeled.

These characteristics are quite intuitive and hold for almost all the 'objects' we deal in our daily life. Let us analyze now some 'logical' traits involved in such a characterization. Firstly, we can say that for every property P, we have that P or its counter property holds (Mittelstaedt's terminology). If we understand this 'counter property' as the negation of P, namely, ¬P, then a form of the excluded middle principle seems to hold: P∨¬P must be true. Secondly, it seems intuitive that we shouldn't accept that contradictory properties hold both at once; so, the contradiction rule $\neg(P\wedge\neg P)$ must also be true. Furthermore, we still have what Mittelstaedt has called the ‘complete determination' of an object (attributed to Kant): "if all possible predicates are taken together with their contradictory opposites then one each pair of contradictory opposites must belong to it" (hence, we could add, each property belongs to it). In other words, it must be also true `the explosion rule', or Duns Scotus Rule, which says that from a contradiction every proposition can be deduced, that is, $(P\wedge\neg P)\rightarrow Q$, for every Q, must be valid. If we continue with these analogies, it seems clear that we shall obtain almost all rules of classical logic. Really, we can say that classical logic was built in complete agreement with O(C). In principle, there is no reason for something which is not an individual according to O(C) not to obey these rules. This is an interesting question, and we have addressed something in this respect in another work (see da Costa and Krause, 2004). But here, at least in the propositional level, we shall admit that the rules of classical propositional logic hold.[3]

The situation becomes more subtle when we consider quantifiers. If we use classical logic (and mathematics) to base our theory, we shall be committed with ‘classical' quantifiers. But due to the standard semantic interpretation (which is part of classical logic), we need identity in the metalanguage. That is, when we say for instance that "There exists an object so and so", and this is true, we mean that there exists a *set* of objects from which we are talking about and that there is an element of this set which is so and so. Furthermore, this object can always be distinguished from any other object of the domain: it is an individual. A similar idea holds for the universal quantifier. That is, *classical* semantics presuppose *classical* set theory (that is, a theory like Zermelo-Fraenkel). Let us recall that *sets*, according to standard set theory, intuitively speaking are collections of *distinguishable* objects, which is in complete agreement with O(C).

So, it is not easy to explain in precise (formal) terms what a physicist intends to say when she speaks that "there exists an electron so and so", or that some quantum systems are entangled, when these entities are taken to be indiscernible from one another. In standard mathematics, strictly speaking there are no absolutely indiscernible objects.[4] That is, we should look with care to the use and nature of quantifiers in quantum logic. Notwithstanding the importance of this point, we shall postpone it to another work (see da Costa et al.,2005).

The important remark is that O(C) is consonant with classical logic and mathematics (set theory) in their main characteristics. But, and what respects quantum systems? As we have seen, quantum ‘objects’ may be completely indiscernible, mainly if we avoid to admit spatial separation as an individuating principle. Thus, how can we treat this situation? To cope with this case, let us summarize, as we have done with O(C), what we mean by *quantum ontology*, O(Q).

Quantum ontology can be traced in contraposition to the characteristics (1)-(5) above. So, let us say that:

(1) Quantum objects are not continuants. Even if we suppose that they have trajectories

(perhaps in a Bohmiam sense, yet in Bohm's theory they are individuals), when they form an entangled system we cannot even more distinguish them from one another any more. That is, there is no apparent contradiction in supposing that quantum systems can be counted as more than one, as in a Bose-Einstein condensate. The fact is that once permuted, the resulting system is exactly the same, for all physical purposes, to the previous one we had before the permutation.

(2) As in O(C), they are objects of predication, and may bear properties, like mass, charge and angular momentum.

(3) They are cannot be individualisable in the sense that this individuality remains forever (or at least for a suitable space of time). Although we can distinguish, say, between two electrons when they are sufficiently apart, this individuality is a *mock* one (to use Toraldo di Francia's words –see his (1985)). In other words, there is no `something' which confers then an identity; there is no Lockean substratum. Redhead and Teller have discussed this point (see Redhead and Teller, op.cit).

(4) We may suppose that they have no identity. This point was emphasized mainly by Schrödinger. In a series of public lectures given at the Institute for Advanced Studies in Dublin in 1950, subsequently published as *Science and Humanism* (Schrödinger, 1951), he writes that, in the face of quantum physics,

> "… we have (…) been compelled to dismiss the idea that (…) a particle is an individual entity which retains its 'sameness' forever. Quite the contrary, we are now obliged to assert that the ultimate constituents of matter have no `sameness' at all. (…) I beg to emphasize this and I beg you to believe it: It is not a question of our being able to ascertain the identity in some instances and not being able to do so in others. It is beyond doubt that the question of `sameness', of identity, really and truly has no meaning." (op.cit.: 17)

(5) Quantum objects cannot be counted, named, tagged, labeled. Suppose that we have two quite similar classical objects, say two identical twins, Peter and Mark. Suppose yet that even their mother cannot differentiate them. But of course even the mother will recognize the difference between the situation where Peter is in the kitchen and Mark is in the garage and the other way around. That is, classical objects, although quite similar, *are* individuals. Permutations *are* observable and produce distinct arrangements. In fact, we can always suppose that Peter dresses a blue T-shirt, while Mark dresses a red one. Identical twins can be labeled, named ('Peter' and 'Mark'). This situation can be made perfectly clear if we say that they obey Maxwell-Bolzmann statistics. But, as concerning quantum objects, the situation is completely different. As Schrödinger said, "you cannot mark an electron, you cannot paint it red" (Schrödinger, 1953). In other words, there is no sense in labeling an electron. But even so we can suppose (as in quantum theory) there are collections with *several* quantum objects. Bose-Einstein condensates provide a nice example.

So, while several authors sustain that the intuitive idea of numerical distinctness does not apply to quantum objects, we would like to say that *distincness* perhaps does not apply but that the idea of *numerical* objects does, although their quantity may vary in time (as in relativistic quantum theory).

These characteristics, here only sketched, entail that the use of the mentioned ‘classical languages’ of logic and mathematics may be inadequate to treat some questions involving quantum objects.[5] Schrödinger had also remarked this point, saying that "[w]e have taken over from previous theory [that is, classical mechanics and O(C)] the idea of a particle and all the technical language concerning it. This idea is inadequate. It constantly drives our mind to ask for information which has obviously no significance. Its imaginative structure exhibits features which are alien to the real particle" (Schrödinger, 1998: 202). It is beyond doubt that by `real particle' he is referring to quantum objects.

The proposal for considering a *new ontology* involving quantum physics has been admitted by several authors, for instance, Bitbol (1996) and Levy-Leblond (2003), but none of them questions classical logic or (mainly) set theory. But there are exceptions. For instance, in addressing the first problem to the ‘new list’ of Problems of Present Day Mathematics, in a meeting sponsored by the American Mathematical Society to discuss the famous Hilbert’s 23 Problems of Mathematics, Yuri Manin asked for a way of considering ‘sets’ (his quotation marks) of indistinguishable objects, as those provided by quantum mechanics (Manin 1976, see French and Krause, op. cit. for an exposition of Manin’s problem and its consequences). Another example is given in M.L. Dalla Chiara and G. Toraldo di Francia’s works; in short, they have questioned the use of standard extensional set theories in this field (Dalla Chiara and Toraldo di Francia, 1993; see also French and Krause, op.cit.).

Our approach follows Schrödinger’s hints about the lack of identity for quantum objects. Intuitively, this means that there is no meaning to speak, within the framework we are considering, neither about the identity nor about the difference, say, between electrons. For all physical purposes, ‘two’ electrons are perfectly alike, but of course they are not ‘the same entity’. According to Schrödinger, as we have seen before, this kind of talk is simply meaningless. We have discussed Schrödinger’s ideas and justified this move in several papers (see for instance French and Krause, op.cit.: Chap.3 for a summary), so as proposed “Schrödinger Logics” to cope with them formally (da Costa and Krause, 1994; French and Krause, op.cit.: Chap.8). In brief, our systems are built so that expressions like x=y are not well formed when x and y denote such entities. We will not enter in this formal discussion here, but next we shall resume the main ideas of our approach just to show how it can provide a way of looking at ‘separate’ but non-individual objects. .

A short account on identity in classical logic

Let us recall in brief how classical logic (and standard mathematics, that is, that one built on a set theory like Zermelo-Fraenkel, ZF) encompasses the concept of identity. A fundamental role is played by the Principle of the Identity of Indiscernibles, which holds in this logic in some way. In first order languages, with = as a primitive symbol (in some first order languages, identity can be defined), the axioms are those of equality, namely, (1) (reflexivity) $\forall x(x=x)$, and (2) (substitutivity) $\forall x \forall y(x=y \rightarrow (F(x) \rightarrow F(y))$, where F(x) is a formula in which x is free and F(y) arises from F(x) by the substitution of y for free occurrences of x. The intended interpretation is so that the equality relation should stand for the diagonal of the domain D, that is, it should correspond to the set $\Delta = \{<x, x> : x \in D\}$. But, as it is well known, these axioms do not characterize the diagonal, for they do not distinguish between individuals of D and certain equivalence classes of elements of D (for details, see Krause and Coelho, 2005). To rightly define identity, we need higher-order languages. For instance, in second order logic, we can define identity by means of the so-called Leibniz Law, namely,

$$\text{(LL)} \qquad x = y =_{def} \forall F\ (F(x) \rightarrow F(y)),$$

where x and y denote individuals and F is a variable ranging over the collection of properties of these individuals. This expression shows that identity is defined by means of indistinguishability. In other words, to fix a terminology, we say that x and y are identical, in symbols, x=y, when there are no two individuals, but only one, which can be denoted indifferently by either x or y, and that they are indistinguishable (or indiscernible) when agree in all their properties, that is, $\forall F\ (F(x) \rightarrow F(y))$. The validity of this rule shows what we have said above, namely, that in standard logic there are no indistinguishable individuals: indistinguishable individuals are the very same object.

In set theory (hence in standard mathematics), the definition can be written, say in ZF (but similar situations occur in other theories like von Neumann-Bernays-Gödel or in Quine's NF), as follows: $x = y =_{def} \forall z\ (z \in x \leftrightarrow z \in y)$ (a slight modification is needed if the theory admits *Urelemente*, but the same idea remains). This entails that all objects in a set theory like ZF (that is, the objects that belong to the standard model of ZF) are *individuals*, yet this affirmative needs an adequate qualification; more technically, we can say that the structure $<V, \in>$, the standard model of ZF, is rigid, in the sense that its only automorphism is the identity function. This entails that an element (set) *a* whatever can be always distinguished from any other, since it is the only element that belongs to the unitary set $\{a\}$, which stands (extensionally) for the 'property' "being identical with *a*" (for details, see Krause and Coelho, op.cit.).

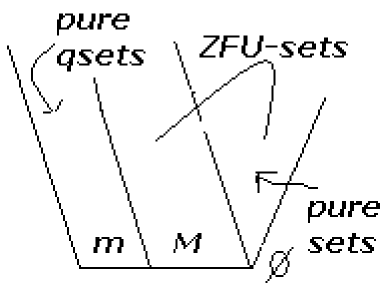
All of this entail that if we have the 'classical theory of identity', there is no way of considering indistinguishable quanta. Physics use some 'mathematical tricks' which of course make the theory work, but at the expense of lots of philosophical difficulties. These 'tricks' are presented in various alternative ways under the general name of a certain Indistinguishability Postulate, IP (see Redhead and Teller, op.cit., French and Krause, op.cit.). Intuitively speaking, IP asserts that permutations of indistinguishable quanta (that is, those agreeing in all their intrinsic properties) are not regarded as observable (a typical case is Bose-Einstein statistics). This is equivalent to postulate that only symmetric and anti-symmetric functions (or vectors) have physical meaning. But the formalism, as extensively discussed by Redhead and Teller, op.cit., begins by attaching them labels, say by means of coordinates in a suitable basis, and then it is assumed that only symmetric and anti-symmetric vectors (or functions) are the relevant ones. This makes the initial labeling otiose but, in our opinion, masks the individuality initially assumed. Using 'classical' languages, there is no other way of expressing indistinguishability. We *need* to make use of these subterfuges: our usual languages are *objectual* (to use Toraldo di Francia's words

–see his (1985)). Notwithstanding, as Manin has suggested, we may ask for a new language to deal with collections of such indistinguishable things. This is the language of *quasi-set* theory.

Using quasi-sets

The idea of having collections of indistinguishable objects may be viewed as follows: such collections should have a cardinal, but not an associated ordinal. Intuitively, this means that we can know how many objects there are in the collection, but we can't name them, label them, order them. The absence of the concept of ordinal associated to the collection may help in fixing the idea that we do have indistinguishable objects. In fact, "countability" is precisely what is problematic about quantum entities. Teller, for example, has insisted that quanta cannot be counted but only *aggregated*, and invokes an analogy (previously deployed by Mary Hesse and Schrödinger) of money in a bank to exemplify the issue (see French and Krause, op.cit.). In short, although I can say that I have a hundred dollars in my bank account, there is no sense in trying to identify these dollars as individuals. This idea is consonant with what happens in certain physical situations, for nothing can distinguish, say, among the neutrons that belong to a nucleus of a Lithium-7 atom (by the way, this atom has 2 neutrons in the nucleus).

However, to think of a 'set' which has cardinal but not an ordinal is quite problematic, if these concepts are understood as usual. The reason is that, in standard set theories, a cardinal is a particular ordinal (more precisely, a cardinal is an ordinal which is not order-isomorphic to any other smaller ordinal). Of course we could think of some alternative to solve the issue, but meanwhile we have yet another problem to consider, namely, to give sense to a concept of indistinguishability which does not "collapse" into identity, as in standard mathematics. In quasi-set theory, this is achieved by using a primitive relation ≡ of indistinguishability, which is reflexive, symmetric and transitive. The language still has three primitive unary predicates, *m, M,* and *Z.* The first two denote *Urelemente*: *m*(x) says that x is a micro-atom, *M*(x) says that x is a macro-atom (the postulates will entail that these objects have all the properties of standard *Urelemente* of ZFU –Zermelo-Fraenkel with *Urelemente*). *Z*(x) says that x is a 'set', which according to the postulates acts as a *copy* of a set of ZFU. If x and y are m-atoms, then the expression x=y is not a formula (this follows from the definition of equality), but they can be indistinguishable (that is, x≡y may be true). Quasi-sets (qsets) are obtained by suitable ZFU-like postulates. "Pure" qsets have only m-atoms as elements. If these m-atoms are all indistinguishable (that is, partake the relation ≡), then since we cannot say neither that x=y nor that x≠y (the negation of the former), there is a sense in saying that m-atoms can be indistinguishable, but not identical. The figure below exemplifies the quasi-set universe.



The qset universe

The language still has a primitive unary functional symbol *qc*; intuitively speaking, *qc*(x) is the quasi-cardinal of the qset x. The postulates entail that *qc*(x) coincides with the cardinal of x (defined in the usual way) when x is a copy of a ZFU-set. But since no order relation can be defined over a pure qset, no ordinal can be associated to these collections, although they have a quasi-cardinal. We shall not give the full axiomatics here (but see Krause, 1992, French and Krause, op.cit: Chap.7).

Important to mention that formally we have a way of expressing that a collection of objects may have a cardinal (its quasi-cardinal) without an associated ordinal and, in addition, we can talk about indistinguishable but not identical objects in our language.

Indistinguishable elements of a qset exist within the framework provided by quasi-set theory, once we can quantify over them. Their 'real' existence in the 'real world' is an *external* question, to use Carnap's terminology (Carnap, 1950). So, this theory can be useful for providing a mathematical framework for semantical analyses of the languages of quantum physics. Furthermore, there is a theorem of quasi-set theory which says (intuitively speaking) that if we 'permute' (by adequate quasi-set operations) an m-atom of a quasi-set by an indistinguishable one which eventually does not belong to it, the resulting qset is by its turn indistinguishable from the original qset (the relation of indistinguishability applies to qsets as well). This has interesting consequences, for we have a formal way of expressing the unobservability of permutations in terms of collections of objects.

The full development of quasi-set theory cannot be done here, to which we suggest the mentioned references. In the next section, we shall outline how quasi-set theory can help us in expressing a form of separability.

Separable non-individuals

Let us suppose, in the sense informally delineated above, that we have a qset A with quasi-cardinal *n* (a natural number) and that its elements are all indistinguishable from one another. Since *n* is a finite cardinal, we can say that (informally) there are *n* elements in A. But it results from the postulates of the theory that no partial ordering relation (in the intuitive sense of $\leq$) can be defined on A (since the basic postulates to be proven demand equality, for instance for expressing anti-symmetry). As concerning *strict* orderings (that is, in the intuitive sense of $<$), the reader may be rightly in doubt, for it demands only to be irreflexive (that is, it is never the case that $x < x$) and transitive, and so equality is never used. This is right, but while in ZF (say) we have that $\langle a,b,c\rangle \neq \langle b,a,c\rangle$ if $a \neq b$, this does not happen in quasi-set theory. In this theory, any permutation of indistinguishable elements leads us to an indistinguishable quasi-set, and the theory cannot offer any kind of distinction between them. So, even if we suppose a strict order on a qset, this order will be not 'detectable' by the theory, and everything will continue to be as if it simply did not exist.

These formal results have interesting philosophical consequences. First of all, let us say that we are not suggesting that Einstein would agree with the existence of objects which don't have identity. In fact, we can say that Einstein has put, yet indirectly, the question of individuality, saying that it would legitimate that the theory describes individual systems. But, in being so, entanglement would correspond to non-separability, as the EPR argument has shown, and this would be the reason why, according to him, quantum mechanics does not represent individual systems, but only statistical ensembles of them (Paty, 2005). Notwithstanding, even he (I suppose) would not deny that we need to consider entanglement for, as Schrödinger said, entanglement is to viewed "(…) not as one, but as *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought" (*apud* Ghirardi, 2005: 165). The difference is that, to Einstein, if the quantum theory could produce such incredibly bizarre effects (non-separability, non-locality), then it had to be invalid as it is, needing to be completed in some sense.

I have never hear either that he wrote on Schrödinger's ideas mentioned above, which appeared some years before his death, or about what could be his opinion on this point. Notwithstanding his particular credo, due to the formal possibility of the existence of such collections (of indistinguishable objects) within a mathematical framework, we may guess that today we regard as licit to think of *separated* objects so as to think of collections of them having cardinalities (more precisely, quasi-cardinalities) greater than one. Furthermore, we should realize that to admit indistinguishable objects does not entail that we do not have separability in a sense. Really, we can argue on this point according to two perspectives:

(1) The first is a metaphysics which assumes that, despite we cannot distinguish between indistinguishable objects *within* the theory, this could be done *from outside* of the theory. I guess that it is something in this sense that is implicit in van Fraassen's following remarks:

> "(..) if two particles are of the same kind, and have the same state motion, nothing in the quantum-mechanical description distinguishes them. Yet this is possible." (van Fraassen,1998).
>
> "A claim of individuation is a completeness claim for a family of predicates; the PII [Leibniz' Principle of the Identity of Indiscernibles] entails that there must always be some family of predicates which individuates. Thus in this perspective, [the] boson case provides the challenge to PII. But this challenge can be met, because it is consistent to postulate a characteristic (such as genidentity) which *is not described in the language*, and individuates without affecting empirical predictions." (van Fraassen, 1985, my italics).

This point of view is consonant with a metaphysics of individuals, and can be sustained by his well known modal interpretation, where the distinction between stated and events gives the way for preserving PII *from the outside*. Another alternative would be to impose some restrictions on the states available to quantum objects and to the observables we should consider.[6] This interesting remark brings to the field of empirical theories a problem which is well known in the realm of the foundations of set theory, namely, that we can look at a certain object or concept *inside* a certain model so as from *outside* the model. For instance, we know that the real numbers are not denumerable in the standard model of first-order ZF, but must be denumerable in the denumerable model, which exists due to the Löwenheim-Skolem theorem (supposing ZF consistent). In our case, following this first alternative, we may say that, *within* quantum mechanics, no distinction can be made for indistinguishable objects, but perhaps this can be done from the *outside*. Of course this makes sense only if we can say in precise terms what is to be understood by a 'model' of quantum mechanics. This is a topic to be further pursued (see Bueno, Krause and Pessoa, 2005).

(2) The second way of admitting separability is to suppose that indistinguishable quantum objects are really indistinguishable and that *even in mente Dei* they cannot be made distinct from one another (this expression was taken from Dalla Chiara and Toraldo di Francia, 1993). That is, we have here a metaphysics of non-individuals. But even being completely indistinguishable, as are for instance the neutrons in a Barium nucleus, we talk of *them* (in the plural), mention *them* as forming certain swarms and so on. So, how to use such a vocabulary? If our words don't have a counterpart in reality (for which we would need to prove that these entities do exist, which is something to be done by physicists), it would be interesting to have a mathematical stuff where the semantics for such a language could be developed within the spirit of the metaphysics itself. That is, when we say, for instance, that certain atoms or other quantum objects are entangled, say in a Bose-Einstein condensate, what are we really talking about? It seems clear that we need a mathematical way of expressing these objects and collections of them. In other works, our *framework* (to use again Carnap's terminology –see Carnap, 1950) should accommodate indistinguishable objects.

Any metaphysics we chose (and we do not need to decide for one of them here) entails that we have *separated* entities of a sort. The cardinalities of the collections of them, being greater than one, shows that this is possible to sustain such a view if we use an adequate mathematical device which do not handle mathematical *ad hoc* devices. The core of this view, that is, the development of quantum mechanics within such a basis, will be pursued further in other works. This paper aimed to be only an initial step in this direction.

Notes

1. We should be very careful in saying that Einstein was a realist, for it depends on the meaning we give to this term. Don Howard calls our attention to a letter from Einstein to Eduard Study, in which he says that "I concede that the natural sciences concern the 'real', but I am still not a realist" (Howard [2004]).
2. Locke termed it something "I don't know what" (Locke, 1994, Chap. XXIII, Book II).
3. From another point of view, we know that classical mechanics can be associated to classical logic by means of a Boolean Algebra, as shown in Jauch 1967.
4. We shall not address to this question here either. But see French and Krause, op.cit., where a study in this direction is done. In short, classical logic and standard mathematics (that is, that one built, say, in Zermelo-Fraenkel) admit a form of Leibniz Principle of the Identity of Indiscernibles: indistinguishable (or indiscernible) objects are the very same object.
5. Let us remark that we are not saying that these 'classical' languages are not suitable for developing quantum mechanics. As it is well known, the most well known formulations of this theory are made using them and physics does *work*. But, for *some* problems, mainly of philosophical nature, for instance to

accommodate non-individual quanta, they can be used only if we admit the existence of some additional symmetry principles like some form of the so called Indistinguishability Postulate (for details on this point, see French and Krause [op.cit.], Redhead and Teller, op.cit). But in our opinion this is a trick, and we should look for a right formalism which would be able to accommodate a metaphysics of non-individuals, just to provide another perspective to this field, perhaps with some philosophical gain.

6. This alternative package was firstly presented by S. French and M. Readhead in their (1988). See French and Krause, op.cit., Chap. 5.

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