Quantum Vagueness*

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Abstract

It has been suggested that quantum particles are genuinely *vague* objects (Lowe 1994a). The present work explores this suggestion in terms of the various metaphysical packages that are available for describing such particles. The formal frameworks underpinning such packages are outlined and issues of identity and reference are considered from this overall perspective. In doing so we hope to illuminate the diverse ways in which vagueness can arise in the quantum context.

1 Introduction

Are there vague objects? Some might say that the answer is a straightforward 'yes': mountains, heaps of sand, heads of hair, even ships can all be taken to be vague. However, it is generally agreed that in such cases the vagueness is situated at the level of concepts, or of language in general; it is the terms 'ship' or 'heap' which are taken to be vague, not the objects themselves. The issue is then whether this conceptual or linguistic vagueness is simply due to ignorance and will evaporate once further facts are known or whether it is ineliminable, requiring a change in our semantic framework in general and the abandonment of bivalence in particular. The idea that the objects themselves might be vague is typically regarded as unintelligible:

> "The only intelligible account of vagueness locates it in our thought and language. The reason it's vague where the outback begins is not

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> that there's this thing, the outback, with imprecise borders; rather there are many things, with different borders, and nobody has been fool enough to try and enforce a choice of one of them as the official referent of the word 'outback'. Vagueness is semantic indecision." (Lewis 1986, p. 212)

A vague object, then, can be regarded as one which has imprecise properties and the unintelligibility of such a notion can be reinforced by appeal to the 'Principle of Ontological Determinacy', attributed by Geach to McTaggart:

> "Nothing can have a determinable characteristic without having it in a perfectly determinate form. For example, nothing can have shape without having a perfectly determinate shape, or size without having a perfectly determinate size, or colour without having a perfectly determinate colour. Vagueness and fuzziness can infect only our descriptions, not the actual things we describe" (Geach 1979, p. 55).[1]

If one accepts that the notion of vague existence is likewise unintelligible (Lewis *ibid.*, p. 213), then there would appear to be no way of getting to grips with the idea that an object itself could be vague.

However, there is a third possibility which offers a means of characterising a sense of ontic vagueness, in terms, not of properties or existence, but in that of individuality and self-identity. The way forward has been plotted by Lowe who argues that identity statements represented by '$a = b$' (see Williamson 1994) are 'ontically' indeterminate in the quantum mechanical context (Lowe 1994).[2]

We shall begin by outlining Lowe's argument and then move on to consider and reject Noonan's well known response. The argument itself is presented within the framework of an approach to the issue of identity in quantum mechanics developed by French and Redhead (1988). Within this approach it is argued that quantum mechanics supports a diversity of metaphysical 'packages' when it comes to this issue and the bulk of our paper is taken up with exploring the possibilities for ontic vagueness within the context of each of these 'packages'.[3]

[1] Expressed thus, the Principle immediately seems problematic given the results established in the foundations of quantum mechanics over the last three decades or so. Is it the case that nothing can have spin, for example, without having a perfectly determinate spin? According to Bell's Theorem, as is well known, determinate (in the sense of predetermined prior to measurement) projections for spins of particles in certain situations are incompatible with the correlations predicted by quantum mechanics. Generalising, and without going into further details, one may legitimately suggest that the sequence of famous no-hidden-variables results, from Gleason and Kochen-Specker through to Bell and beyond all point to a kind ontic indeterminacy. Our intention is to approach this issue from a different, but more fundamental, direction. At the heart of the issue, as we shall see, is the notion of 'entanglement' involved in these situations.

[2] In his Aristotelian Society debate with Williamson on vagueness, Peter Simons dismisses the example of quantum indeterminacy because he is concerned to defend the possibility of semantic vagueness even if the world were perfectly determinate (Simons 1992).

[3] Thus the present paper is a further development of the ideas sketched in (French and Krause 1995).

One such position is that quantum particles may be regarded as individuals and it is this view that Lowe adopts as the basis for his claim. Exploring this line further we emphasise the role of 'entanglement' in Lowe's argument and suggest that vagueness can then be understood in terms of Teller's notion of 'non-supervenient' relations holding between the particles (Teller 1986, 1989). This suggestion distinguishes our approach from other responses to Lowe's argument which have tended to concentrate on the diachronic aspect. We shall briefly consider such responses before moving on to the alternative metaphysical 'package' regarding particle individuality.

Historically, this was the 'Received View' for many years and takes the particles to be 'non-individuals', in some sense. We argue that this notion can be understood in terms of a lack of self-identity and we defend the intelligibillity of talk of objects without well defined identity conditions in this context. This idea has been formally expressed using the technical apparatus of 'quasi-set' theory (Krause 1992, 1996; Krause and French 1995) and here we indicate how this technical machinery might be put to use to give a clearer perspective on 'quantum vagueness'.

2 Indistinguishability and Individuality

In this section we recap the discussions of French and Redhead (1988) and French (1989a and c). Both classical and quantum particles of the same species or 'kind'[4] are said to be indistinguishable, in the sense of sharing the same set of intrinsic, state-independent properties, such as rest-mass, charge, spin, etc.[5] Nevertheless, there is a clear difference in their collective behaviour, as described by classical and quantum statistics respectively. In classical statistical mechanics a permutation of particles between states is counted as giving rise to a distinct arrangement or complexion, whereas in quantum statistics it is not. It is then argued that, in the classical realm at least, this implies that the particles are individuals and, since they are indistinguishable in the above sense, this individuality must be grounded in something over and above their (non-spatio-temporal) intrinsic properties (Post 1963, French 1989a).[6]

In quantum statistics, on the other hand, the so-called 'Indistinguishability

[4]We shall not be concerned here with the issue of whether such species count as 'natural' kinds or not.

[5]See, for example, Jauch (1966), p. 276. The distinction between intrinsic and extrinsic properties is, of course, much discussed in the philosophical literature; for a recent discussion see Butterfield (1993).

[6]Post referred to this as 'Transcendental Individuality' and French noted that this could be interpreted either in terms of some form of 'Lockean substratum' or in terms of a correspondence with the points of space-time ('Space-Time Individuality'). Redhead and Teller, on the other hand, understand it in terms of 'primitive thisness' or haecceity (Redhead and Teller 1991, 1992; Teller 1995). In response to these latter developments, Huggett has argued that classical particles can be understood as individuals in a sense other than that of 'primitive thisness', where this other sense involves a form of 'exclusion' in terms of which the particles can still be thought of as following distinct spatio-temporal trajectories (Huggett 1995; private e-mail). This seems to match French's notion of 'Space-Time Individuality'.

Postulate' holds. This states that,

If a particle permutation P is applied to any state function for an assembly of particles, then there is no way of distinguishing the resulting permuted state function from the original unpermuted one by means of any observation at any time.[7]

In other words, permutations of quantum particles are not counted as giving rise to new arrangements. It is this which gives rise to Bose-Einstein, Fermi-Dirac and 'para-'statistics.

One way of interpreting this postulate is to claim that it implies that quantum particles cannot be regarded as individuals in some sense, where this is taken as meshing nicely with the metaphysics of quantum field theory (Post *op. cit.*; Redhead and Teller 1992; Teller 1995). We shall return to this claim below. Alternatively it has been argued that the particles can still be regarded as individuals, with the non-classical counting of quantum statistics understood as due to restrictions imposed by the above Postulate on the sets of states accessible to the particles (French 1989a; French and Redhead 1988; Redhead and Teller 1992). The particle labels therefore have denotational significance on this view –to discard that significance because we cannot tell which label attaches to which particle after a permutation would be to accede to a crude and unwarranted form of positivism.[8] The details have been extensively discussed elsewhere; what is important here is to note that it is this latter account which provides the framework for Lowe's argument that quantum objects are vague objects (Lowe 1994a).

3 Vague Identity and Quantum Indeterminacy

Lowe considers a situation in which a free electron a is captured by an atom to form a negative ion which then emits an electron labeled b and notes that,

> "... according to currently accepted quantum mechanical principles there may simply be no objective fact of the matter as to whether or not *a* is identical with *b*. It should be emphasised that what is being proposed here is not merely that we have no way of telling whether or not *a* and *b* are identical, which would imply only an epistemic indeterminacy. It is well known that the sort of indeterminacy presupposed by orthodox interpretations of quantum theory is more than merely epistemic – it is ontic.' (p. 110)

At this juncture it is important to emphasise that the basis of indeterminacy is essentially that quantum particles may enter into 'superposed' or 'entangled'

[7]See, for example, French and Redhead *op. cit.*; van Fraassen 1991 p. 381.

[8]Of course, in the context of such a view the status of Leibniz's Principle of the Identity of Indiscernibles becomes highly problematic (French and Redhead 1988; French 1989b; Dalla Chiara and Toraldo di Francia 1995). We shall not dwell on that issue here, except to recall that both bosons and fermions receive equal treatment on this view (French and Redhead *op. cit.*). Efforts to break this symmetric treatment typically involve the attribution of 'empirically superfluous' factors; see van Fraassen 1991.

states in which the number present is determinate, since –crucially– they may be regarded as individuals, but the identity of any with a given particle is not: it is because electrons a and b are, prior to a measurement, regarded as being in such an entangled state (together with the other electrons of the atom, of course) that their identity is indeterminate. We shall return to this point shortly.

Now, as Lowe notes, there is a well known argument against the possibility of vague objects, given by Evans and, independently, Salmon (Evans 1978; Salmon 1982, pp. 243-246). In a nutshell the argument is as follows (here we are simply following Lowe): Suppose for reductio, that it is indeterminate whether $a = b$, where 'a' and 'b' are precise designators, in some sense.[9] Then b definitely has the property that it is indeterminate whether it is identical with a, but a definitely lacks this property (since $a = a$ is surely not indeterminate), hence it is false that $a = b$. The upshot is that if '$a = b$' is indeed indeterminate in truth value then either 'a' or 'b' or both must be an imprecise designator.

We shall also give the argument in its full form, as we wish to highlight the various assumptions that are made (again we follow Lowe and use '$\bigtriangledown p$' to mean 'it is indeterminate whether p').

Assumption 1: 'a' and 'b' are precise designators:[10]

$$\begin{array}{rll} & (1) & \bigtriangledown(a = b) \\ \Rightarrow & (2) & \widehat{x}[\bigtriangledown(x = a)]b \end{array}$$

Assumption 2: '$a = a$' is determinately true:

$$\begin{array}{rll} & (3) & \neg\bigtriangledown(a = a) \\ \Rightarrow & (4) & \neg\widehat{x}[\bigtriangledown(x = a)]a \end{array}$$

Assumption 3: the existence of a predicate true of a whose negation is true of b suffices for the truth of '$\neg(a = b)$'

$$\Rightarrow \quad (5) \quad \neg(a = b),$$

contradicting the initial premise that '$a = b$' is indeterminate in truth value.

Lowe's response to this argument is very interesting: he rejects the move from (3) to (4). His grounds for doing so are that if we allow that b possesses the property expressed by $\widehat{x}[\bigtriangledown(x = a)]$ then by parity of reasoning we must say that a possesses the symmetrical property expressed by $\widehat{x}[\bigtriangledown(x = b)]$. But if it is indeterminate whether object a is identical with object b (recall *Assumption 1*,

[9] The use of the term 'precise designator' indicates an analogy with the way in which the term 'rigid designator' functions in possible worlds semantics; see Lewis (1988).

[10] In equation (2) below, Lewis (1988) glosses this as 'b possesses the property of its being indeterminate whether it is identical with a'.

that 'a' and 'b' are precise designators) then the property expressed by $\widehat{x}[\triangledown(x = a)]$ cannot be determinately distinct from the symmetrical property expressed by $\widehat{x}[\triangledown(x = b)]$, for these 'two' properties differ only by permutation of a and b. Hence the possession by b of the property expressed by $\widehat{x}[\triangledown(x = a)]$ cannot differentiate b determinately from a, since that property is not determinately distinct from a property which is possessed by a. Hence we are not entitled, solely on the basis of the logical truth (3) to assert (4), which denies that a possesses the property expressed by $\widehat{x}[\triangledown(x = a)]$.

In a response to Lowe's paper, Noonan has attempted to reassert the force of the Evans/Salmon argument by re-construing it in terms of properties that are not identity involving (Noonan 1995). Thus he presents two case studies to illustrate his reformulation of the argument, and in each it is again supposed, for reductio, that identity is indeterminate. We can paraphrase these cases as follows:

Case 1 (diachronic identity): brain transplants.

Pauli's brain is put in Steve Martin's body, where Pauli is fat and Martin is thin. After the transplant Paultin is thin, but it is indeterminate whether Pauli is thin after the transplant. Hence Pauli is such that it is indeterminate whether he is thin after the transplant, but Paultin is not such that it is indeterminate whether he is thin after the transplant. Consequently, by application of Leibniz's Law, we can conclude that Pauli is not Paultin, contradicting the initial assumption that the identity is indeterminate. The important point to note is that the property being such that it is indeterminate whether a person is thin after the brain transplant is not identity involving.

Case 2 (synchronic identity): connected halls.

The Philosophy Department and the Chemistry School at the University of Leeds are two structures joined by court and bridge in such a way that it is unclear whether they count as two buildings or parts of one building. Suppose that Einstein is lecturing in Philosophy and Pauli in Chemistry. The statement 'the building in which Einstein is lecturing is identical with the building in which Pauli is lecturing' is assumed to be indeterminate in truth value. However, by considering the identity-free property 'being such that it is indeterminate whether it contains Pauli', which the building containing Einstein possesses and the building containing Pauli lacks (again there is an initial assumption that the descriptions are precise designators), we can give a form of the Evans/Salmon argument which leads to the conclusion that, in fact, one or both of the singular terms 'building in which Einstein is lecturing' and 'building in which Pauli is lecturing' must be an imprecise designator.[11]

However, it is easy to see that both of these cases that Noonan presents are disanalogous with the situation in quantum mechanics which forms the basis of Lowe's claim. Case 1 hinges on Pauli and Martin possessing different

[11] Cf. Sainsbury on compositional vagueness (1989).

properties, which is not the case for particles in entangled states. We recall our point above regarding the symmetry of treatment of bosons and fermions: two, or more, bosons, or two, or more, fermions, in an entangled state possess exactly the same monadic and relational properties (or at least all those that can be expressed in terms of the quantum mechanics formalism); they are absolutely indistinguishable.

Case 2 may appear to be analogous to quantum states containing particles but when it comes to the latter, the indeterminacy of 'state 1 = state 2' is, of course, not the issue.[12] Focusing on the particles, the analogue of Noonan's identity-free property would be 'being such that it is indeterminate whether particle a is in state 1' and this is something which both particles possess. Thus, neither of Noonan's 'classical' examples address what is at issue in the quantum case and we must conclude that his attempt to resurrect the Evans/Salmon argument has failed in this context.

Interestingly, after presenting his case studies, Noonan himself writes:

> "Thus a proponent of the view that indeterminacy in identity statements can be the result of ontic indeterminacy, even when the singular terms flanking the sign of identity are precise designators, cannot just rest with Lowe's response to Evans - unless, that is, he is prepared to accept that the only possible cases of indeterminacy in truth-value due solely to ontic indeterminacy are ones in which the relevant indeterminately identical objects are strongly indistinguishable in respect of all their identity-free properties - strongly indistinguishable in the sense that any such property determinately possessed by either is determinately possessed by both. But this seems hardly an attractive position for the proponent of ontic determinacy to occupy ..." (pp. 17-18)

This may seem hardly an attractive position but it is one to which we are forced by quantum mechanics!

4 Quantum Entanglement and Non-Supervenient Relations

Nevertheless, there is something to Noonan's remark that we cannot just rest with Lowe's response, even if we accept that quantum particles are the kind of entity which may possess all identity-free properties in common. Consider: the framework for Lowe's argument is the first of French and Redhead's metaphysical 'packages' according to which indistinguishable particles can still be regarded as individuals. Thus we are entitled to regard 'a' and 'b' as precise designators. But now we have the curious situation in which we have an individual, denotable by an appropriate proper name, such that it seems that this

[12] Indeed, it turns out that self-adjoint functions of the place permutation operators, which operate on the place, or state, labels can be regarded as observables; see French 1989a

individual possesses the property that it is indeterminate whether it is identical to itself. What is it that makes this seem to be the case?

Returning to Lowe's example, we can see that the only other metaphysical elements involved are the *relations* between the electrons. Since the latter are considered to be individuals within this framework, the source of the ontic vagueness cannot lie with their individuality; hence our focus must shift to the relations. From the point of view of physics, these are the 'entangled' relations characteristic of quantum mechanics under its orthodox interpretation.[13] Teller has argued that such relations are 'inherent' or 'non-supervenient', a view which he calls 'relational holism'. His claim is that if, in contrast, they are taken to supervene on 'hidden' intrinsic properties of the particles, (a view he calls 'particularism'), Bell's inequalities result, and if they supervene on 'extrinsic' properties the incorrect quantum state is obtained (Teller 1986).[14]

'Relational holism' might not seem so radical a metaphysical world view following Lewis's analysis of spatio-temporal relations as non-supervenient or, as he calls them, 'external'.[15] However, there are clear differences between spatio-temporal relations and those exemplified by entangled states.[16] First of all, the latter are non-discriminating, in the sense that two particles in such a state are exactly alike so far as these relations are concerned; thus they are not 'analogous' to spatio-temporal relations in Lewis's sense (Lewis 1986, p. 76).[17] Furthermore, there are certain fundamental differences between the two which become manifest when we consider the nature of the subvening properties in each case.

Thus it may appear obvious that spatio-temporal relations do not supervene

[13]It would be interesting to consider whether a form of ontic vagueness could arise under the Bohmian interpretation, for example, but we shall leave this to a future work.

[14]Cleland (*op. cit.*) attributes 'particularism' to Leibniz but Castañeda (1972) claims that it can be traced as far back as Plato.

[15]Thus Butterfield (1993) refers to non-supervenient relations in the quantum context as 'external' in Lewis's sense.

[16]We are grateful to James Ladyman for pointing this out to us.

[17]Lewis himself considers the 'test case' of positive and negative charge being understood as 'natural external relations' and concludes that 'they are very far from discriminating' (*ibid.* p. 77). Since he is concerned here with the unification of possible worlds by 'external interrelatedness', he asks whether two particles in different worlds could stand in such charge-like external relations and finds himself unable to rule out a positive answer. This is what drives him to conclude that the unifying relations must be at least analogically spatio-temporal. A corresponding question might be posed for the quantum 'entangled' relations: could two particles in different worlds stand in such relations? The answer is surely no and this reveals a difference between such relations and Lewis's gerrymandered 'like-chargedness' and 'opposite chargedness'. For two particles in different worlds to stand in such relations, the relations would have to span the gap between worlds. 'Offhand', to use Lewis's word, there would seem to be no difficulty in viewing chargedness this way, but only because of a tacit prior understanding of chargedness as intrinsic. (Suppose we had two worlds, in one of which there was just one electron and in the other there was just one positron. They would be related across worlds by opposite chargedness but of course the behaviour that would form the basis for such an attribution is dispositional and in this case would never be made manifest. The gerrymandered chargedness is only conceivable on the basis of a prior understanding of charge.) Entangled states, however, cannot exist 'between worlds' and thus two particles in different worlds could not be in such a state or be related in such a way. This is related to a further difference between these relations and spatio-temporal ones which we consider below.

on intrinsic properties, such as mass or charge for example. Lewis spells this out by asking us to consider first a hydrogen atom and then the duplicates of the proton and electron which may exhibit a completely different inter-object distance (p. 62; the picture is explicitly classical of course). However, the extent to which spatio-temporal relations are non-supervenient or 'external' depends on both our view of space-time and the nature of individuality. If we were to adopt a relationist view of space-time, together with some form of 'bundle' theory of individuality, for example, then we might argue that spatio-temporal relations are still dependent on the intrinsic properties of the relevant objects, since if these were stripped away there would be no objects and without the latter there would be no spatio-temporal relations. However they are not *determined* by such properties and hence spatio-temporal relations can be described as only 'weakly' non-supervenient (Cleland 1984).

This suggestion rests on the introduction of a possible world in which the intrinsic properties of the particles are stripped away. In the quantum context the situation is quite different. Here we are concerned with, for example, the superposition of two non-relational spin states of the particles in which quantum mechanics does not assign a definite spin to either particle in any direction. If hidden variable states are left out of the picture, then the issue is whether the associated relations are reducible to those represented by 'both particles in the same one-particle state'. It is straightforward to show that they are not (Teller 1986). In this case, we do not need to go to possible worlds, since the relevant states are mutually exclusive in the sense that a system in one cannot simultaneously be in the other (this point was emphasised in French 1989c). Hence, as Teller notes, "... the superposition characterises an independently identifiable property with distinctive experimental implications for the ... system as a whole" (1986, p. 80). Since the properties represented by the superposition are not dependent upon those represented by 'both particles in the same state', then they are not determined by them either and thus they are 'strongly' non-supervenient.

Are there any other non-supervenient relations? Lewis himself considers and rejects several possible candidates before presenting his gerrymandered 'like-' and 'opposite' charged' example (*op. cit.*, p. 77). The fact is, however, spatio-temporal relations aside, the relations which are of interest to physicists all supervene on the intrinsic properties of the particles.[18]

What we have, then, is a metaphysical package consisting of individuals between which there hold relations which are neither determined by nor dependent upon the monadic properties of these individuals (French 1989c). From this metaphysical perspective, they 'veil' the particles in such a way that we cannot assign the latter any determinately distinct identity-free properties. And this 'veiling' of the particles can be understood as giving rise to a form of ontic vagueness, in the following way: it is a central claim of the epistemic view

[18] Weyl suggested that this may have its origin "... within the domain of sense data, which –it is true– can yield but quality and not relation" (Weyl 1963, p. 4). This might explain why relations between 'classical' objects are not regarded as irreducible in Teller's sense, but it should not be taken as binding in the quantum realm.

of vagueness that 'vaguely described facts' supervene on 'precisely describable facts' (Williamson 1994, p. 248; see also pp. 201-204). Such a claim cannot be maintained within the above context: the 'vaguely described fact' concerning the electron's identity in the situation envisaged by Lowe arises from its being in an entangled state. The properties represented by such states do not supervene on either the intrinsic properties of the particles or any set of hidden variables, (this being ruled out by Bell's Theorem). Thus the above 'facts' involving entangled states do not supervene on any facts involving the intrinsic properties of the particles or hidden variables and we have an example of genuine ontic vagueness. [19]

More generally, the epistemic view characterises vagueness as a form of ignorance. However to characterise the above situation in this way obscures a fundamental difference between not knowing which electron is which and not knowing when William of Ockham was born, say (Simons 1992, pp. 166). In the latter case we know what sort of discovery could fill the gap in our knowledge (*ibid.*) but there is no possibility of such a discovery in the case of the former. The ignorance here may be called 'in principle' but calling it that is enough to distinguish it from the epistemic form. Nevertheless, one does not need to be a verificationist to feel uncomfortable with the scholastic metaphysics underpinning this account. In particular, the fact of the matter to which we cannot have access, even in principle, is resolutely metaphysical, consisting, as it does, in the claim that the particles can be regarded as individuals despite being indistinguishable in this strong and non-classical sense. This is the price that must be paid in order to get Lowe's argument –and the consequent ontic vagueness– off the ground in the first place. Of course, one might feel that this price is too high, in which case one might prefer the alternative metaphysical 'package', according to which the particles are 'non-individuals'. In a subsequent section

[19] It is worth noting that Teller's 'relational holism' itself developed out of a concern with vagueness: In his earlier work on continuous quantities (Teller 1979) Teller argued that since point positions and momenta are not represented in Hilbert space, these quantities should be thought of as 'spread out' and imprecise; indeed he gives the analogy of objects with inexact boundaries, such as a puddle (cf. Sainsbury 1989). This work influenced Stairs's well-known 'realist quantum logic programme' which attempted to formalise the idea that

> "... there can be facts about a pair of quantum systems which are, in a clearly specifiable sense, about the whole system but are neither reducible to nor implied by facts about the parts." (Stairs 1984, p. 357)

These facts are 'disjunctive facts' in the sense that certain disjunctions (representing superpositions of states) may be true even though none of their disjuncts are (Stairs 1983, p. 591). Quantum correlations are then explained "... by appeal to the unusual logical relations which quantum systems bear to one another and the way in which these relations bear on probability" (1984, p. 358). Teller himself acknowledges that he developed his view of 'relational holism' on the basis of Stairs's programme. But Stairs also cites Teller's earlier work on continuous quantities and gives as an analogy of a disjunctive fact the example of two neighbouring piles of stones on a pebbly beach, such that there is a particular stone that seems to belong to one pile or the other without clearly belonging to either pile taken individually (Stairs 1983, p. 592). Disjunctive facts are thus vague facts and so, historically, both the so-called 'realist' quantum logic programme and 'relational holism' grew out of a concern with vagueness.

we shall consider the issue whether a form of ontic vagueness can be developed within this alternative framework, but first we shall discuss further criticisms of Lowe's argument, together with his responses.

5 Vagueness and Diachronic Identity

In his own response to Noonan, Lowe argues that the latter's style of argument can in fact be adapted to his quantum mechanical example (Lowe 1997; see also Lowe 1998, pp. 67ff). The reconstruction hinges on Noonan's claim that whereas it is indeterminate whether electron a is emitted when b is, this is obviously not the case for b itself and so a and b differ in their identity-free properties. However, Lowe insists, the invocation of such properties marks a significant shift in direction within the debate, since although in the case of identity statements, as employed in Evans' original proof, the issue of tense can be safely ignored, it cannot when it comes to predications, as in Noonan's argument (Lowe 1997). Crucially, the names assigned to the electrons incorporate an implicit time reference, in the following sense: if t_0 is the time following the capture of electron a and prior to the emission of electron b, when the electrons become entangled and t_1 is some time after the emission of b, then, indeed, at t_1 it is *not* indeterminate whether b has been emitted but *is* indeterminate whether a had been emitted. However, Lowe argues, this does not imply that a and b differ in their properties, as Noonan claims. The point is, it is indeterminate which of the electrons will be emitted and this is true of both a and b.

In a further response, Hawley asks what feature of entanglement could support Lowe's claim (Hawley 1998)? Thus she considers the indeterministic feature of quantum processes (on the standard interpretations) and argues that although it is true that at t_0 it is not *determined* that b will be emitted, this gives us no reason to believe that it is *indeterminate* at t_0 whether it will be emitted. Furthermore, she remarks, the conclusion also does not follow from the assumption (which she grants) that it is indeterminate at t_0 whether a is emitted. If the entangled electron which is not a is called e, then each of a and e is such that it is indeterminate at t_0 whether that electron is later emitted. Now, b is supposed to be one of the entangled electrons, of course and must possess any property shared by a and e, including, presumably, the property of it being indeterminate at t_0 whether it will later be emitted. However, if it is *indeterminate* whether b is identical to a and also *indeterminate* whether it is identical to e, then Hawley argues, there is no reason to suppose that the properties of b are those properties shared by a and e, and thus no reason to suppose that b is such that at t_0 it is indeterminate whether it will be emitted. The consequences of ontic indeterminacy itself undermine Lowe's argument.

Finally, Hawley invites us to consider the temporal symmetry of the situation: if before emission it was indeterminate whether b was going to be emitted and after emission it was determinate that it had been, then although it is determinate before absorption that a will be absorbed, temporal symmetry implies that after absorption it is indeterminate whether a has been absorbed. But now this is problematic. It appears there is no reason to believe that a fact about a –its absorption– becomes indeterminate as time passes. Likewise, there is no reason to believe that a fact about b –its emission– becomes determinate as time passes.

What this response highlights is the central contention –held by Lowe– that quantum particles are such that they can be determinately distinct at one time and yet not at another –later– time. In particular, once entangled, the particles are not determinately distinct, but they may be before their entanglement. In his response to Hawley's critique, however, Lowe acknowledges that once particles have become entangled, they can never thereafter become determinately distinct again (Lowe 1999). His example should now be understood as follows: before absorption, there are two determinately distinct electrons, one, a, which is determinately absorbed and the other, a', which is determinately already inside the atom. Upon absorption entanglement occurs, so the electrons are no longer determinately distinct, even though they are still determinately two in number. Because of the entanglement there is then no fact of the matter as to which electron is subsequently emitted, and if we try to assign the label b to the emitted particle, we cannot do so in a determinate manner, because the definite description 'the emitted electron' does not determinately pick out a unique entity (*ibid.*). On the other hand, the definite description 'the absorbed electron' does determinately pick out a unique entity, at least with respect to the time of absorption, though not with respect to any later time, and hence the asymmetry is restored. Hence Lowe admits here that his original example was misdescribed in taking b to determinately designate a unique electron and consequently the question whether b has a property which a lacks is simply misposed. This yields a coherent example of ontic indeterminacy with regard to identity which, Lowe asserts, undermines Hawley's conclusion that "Talk of particles which are first outside and then within the entangled state is incompatible with talk of ontic indeterminacy" (Hawley *op. cit.*, p. 106)[20].

Before we comment on this exchange, we shall note for completeness the further attack on Lowe mounted by Odrowaz-Sypniewska (Odrowaz-Sypniewska 2001). In particular, she remarks that "[Lowe's] reasons for saying that the entangled electrons are ontically indeterminately identical seem to be that (i) they are determinately two and yet (ii) they are not determinately distinct" (*ibid.*, p. 67). However, she continues, if (i) is true, then it cannot be ontically indeterminate whether the electrons are identical since the claim that two (or more) objects are ontically indeterminately identical is just equivalent to the claim that the number of those objects is indeterminate. Lowe's response, first of all, is that he does not in fact claim that the entangled electrons are ontically indeterminately identical (Lowe 2002). We recall that he has adopted the metaphysical package which regards the particles as individuals and thus, as he emphasises, what we are faced with is an ontic indeterminacy of *diachronic* identity. Secondly –and importantly for our further discussion below– Lowe

[20]In a more recent paper (Lowe 2002), Lowe reverts to his earlier view that b *can* be taken to be a precise designator after the time of emission

elsewhere accepts that determinate countability and determinate identity do not necessarily go hand in hand (Lowe 1998), so that Odrowaz-Sypniewska's argument above begs the question.

Now what are we to make of these exchanges? First of all, let us emphasise once again that Lowe has constructed his argument within the framework that holds the particles to be individuals. Within such a framework, the 'source' of the ontic indeterminacy in the situation envisaged cannot reside in the ontological status of the particles as individuals. Hence it must lie with the nature of the 'entangled' state and Hawley is right to focus on this issue. However, her focus is, in effect, too narrow: while it is true that indeterminism does not lead to indeterminacy in this situation, she does not consider the possibility that the non-supervenient relations holding between the particles in this state effectively prevent us from labelling them or establishing their identity. And of course, as long as the particles remain entangled, these relations will continue to hold, which they will do until a measurement occurs (on the standard interpretation). Thus Lowe is strictly correct in his insistence that we cannot label the electron that is emitted, since it remains in an entangled state. Indeed, one might wonder whether we can even speak of 'emission' strictly speaking! Of course, once we make a measurement we can both label the electron –by pinning it down at a particular spatio-temporal location– and identifying it as having been emitted by the atom. This labelling and identification may then be 'projected back', as it were, to the time of emission itself, but strictly speaking, as we have indicated, this is inappropriate.[21] We shall return to this point below.

Hence, as long as the situation is such that the incoming electron a has not interacted with anything and so is not in an entangled state, then Lowe's asymmetry seems to be restored. However –and this is the crucial point for us– his argument makes it clear that the source of the ontic indeterminacy lies with the entangled state and thus if we are to fully understand this indeterminacy, we must examine the metaphysical nature of this state. Hawley argues that indeterminism cannot do the job, as it were, but in her paper acknowledges that our focus on the individuality of the particles may shed further light on the issue.

Putting it in rather blunt terms, we see the introduction of tense as a red herring in this debate. The issue has to do with the metaphysics of the entangled state, given Lowe's adoption of the particles-as-individuals package, and, as we have argued, if we understand this metaphysics in terms of Teller's notion of non-supervenient relations, we can shed light on the ultimate source of the ontic indeterminacy.

Historically, however, it was not the above metaphysical package that was adopted by most physicists and philosophers commenting on particle identity. Rather, the 'received' view for many years maintained that the particles are, in fact, 'non-individuals' in some sense. We shall now consider how 'quantum vagueness' looks from this perspective.

[21] Of course, it *would* be appropriate if we were to adopt an alternative account –such as Everett's 'many worlds' approach– which takes entanglement to cease when any interaction occurs, not just measurement interactions.

6 Vagueness and Non-Individuality

Let us return to the Indistinguishability Postulate. We recall that in classical statistical mechanics arrangements of particles over states which result from permutations of the particles are counted as distinct and the particles may be regarded as individuals. In quantum statistics it may be argued, by parity of reasoning, that if such arrangements are not regarded as distinct, the particles themselves cannot be regarded as distinct in this sense; they are, in effect, 'non-individuals' (Schrödinger 1952, 1957; Born 1943, pp. 27-29; Post *op. cit.*; French 1989a).

This gives us an alternative metaphysical package which, it is claimed, meshes nicely with that presented by the Fock Space representation of Quantum Field Theory (French and Redhead 1988; Redhead and Teller 1991 and 1992; Teller 1995). This representation does not refer to individual particles at all and an account of many-body systems is given in which particle labels are simply not assigned from the word go. Nevertheless, in discussions of this approach, mention is still made of 'entities' (Redhead and Teller 1991, 1992) or 'quanta' (Teller *op. cit.*); that is, there is still talk of 'objects'. The issue, then, is how we are to understand the notion of 'non-individual' objects, in this context.

A good place to begin is with Lowe's own characterisation of an individual as

> "... an object that is differentiated from others of its kind in such a fashion that it and they are apt to constitute a *countable plurality*, with each member of such a plurality counting for just *one*, a *unit* of its kind" (1994b, p. 536).[22]

This understanding of individuality in terms of distinctness and countability is common, of course. Thus, Russell for example, appears to have had this understanding in mind when he wrote:

> " 'How shall we define the diversity which makes us count objects as two in a census?' We may put the same problem in words that look different, e.g., 'What is meant by 'a particular'?' or 'What sort of objects can have proper names?' " (Russell 1948, p. 292).

We will see shortly how the notion of a 'countable plurality' is ambiguous in this context but let us consider what is problematic about this way of viewing individuality. Castañeda, for example, refers to it as a 'fusion of genuine individuation and differentiation' (1975, p. 134). Similarly, Gracia has argued that distinctness is neither necessary nor sufficient for individuality - not sufficient because we can conceive of distinct universals, for example, and not necessary because we can conceive of a possible world in which there is only one individual

[22] Lowe has recently modified this by substituting '*determinately distinct*' for 'differentiated' (private correspondence).

(1988, p. 34).[23] Such concerns reflect the scholastic distinction between distinguishability, understood as involving more than one object and individuality, understood as pertaining to that object taken by itself. It is through distinguishability or determinate distinctness that we become aware of something as an individual, in a negative fashion as Gracia puts it, but ontologically and positively, as an individual it is one thing or as Suarez said, it has an 'individual unity'.

Having made this distinction, how might we represent this 'individual unity'? What we want is a means of doing so in formal terms which would allow us to get a grip on it in the context of the Evans-Salmon argument and the indeterminacy of identity statements in general. An obvious way forward would be to equate 'individual unity' with self-identity (see, for example, Heidegger 1957, p. 26).[24] Of course, since every being has it, identity is useless for the purposes of individuation or distinguishing one individual from another and hence this approach requires that issues of distinguishability and individuality are kept distinct.[25]

Thus we might characterise a non-individual as an object for which no identity criteria hold. If the theory of identity is taken to be that of classical logic, then for a non-individual it is not the case that '$a = a$'.[26] This allows us to avoid the Evans-Salmon argument and resolve the curious situation Lowe finds himself in: we cannot deny that for quantum particles it is indeterminate whether they are identical to themselves because they are not!

We realise this may seem a bizarre proposal (but whoever said that the metaphysics of quantum objects was *not* bizarre!).[27] And, of course, how we understand the nature of self-identity is crucial. It is important to realize that, in arguing that identity should not be applied to quantum particles, Schrödinger was not merely talking of the impossibility one has of recognizing a particle, once observed, after it has interacted with another of the same species:

> "I beg to emphasize this and I beg you to believe it: It is not a question of being able to ascertain the identity in some instances and not being able to do so in others. It is beyond doubt that the

[23]This claim is itself problematic. If we insist that possible worlds not be introduced devoid of space-time structure, then such an individual may be regarded as 'determinately distinct' from this structure. Of course this raises issues concerning the absolute-relational debate and the individuality of space-times points which we shall not discuss here.

[24]Auyang, whose concerns match ours, writes: "Identity does not say anything beyond one thing; rather, it discloses the meaning of *being an entity*, and the disclosure signifies our primordial understanding" (1995, p. 125). Adams famously prefers the term 'thisness', taken from Duns Scotus, where "A thisness is the property of being identical with a certain particular individual" (1979, p. 7). The latter term has subsequently been adopted by Teller (1995, 1998).

[25]Lowe himself has acknowledged that an entity may be determinately self-identical but not determinately distinct, for example (private correspondence).

[26]Hesse also repeatedly refers to quantum particles as lacking self-identity, writing that "[w]e are unable to identify individual electrons, hence it is meaningless to speak of the self-identity of electrons ..." (1970, p. 50).

[27]"I get much more consternation and claims of unintelligibility than concrete argument for the inviolable status of the applicability of strict identity"(Teller 1998).

question of 'sameness', of identity, really and truly has no meaning." (Schrödinger 1952, pp. 17-18)

In other words, the problem runs deeper than simply questioning diachronic identity (which is a problem in itself of course).[28]

Wittgenstein famously rejected Russell's definition of identity by means of the identity of indiscernibles and insisted that identity should not be regarded as a relation between objects.[29] The history of philosophy aside, it would be a mistake to view identity as a property or relation like other relations. Barcan Marcus errs in writing that, "Individuals must be there before they enter into relations, even relations of self-identity" (1993, p. 20). Talk of 'entering into' is surely misplaced in this case: it seems apt in the case of an individual 'entering into' a relation, such as parenthood for example, but not in the case of an individual entering into the relation of identity with itself. In the former case the relation can be formed only when there is more than one individual; the establishment of the relation is conceptually posterior to the existence of the relevant individuals. In the latter case, however, the existence of the individual and the establishment of self-identity are conceptually on a par in that we cannot envisage the possibility of one without the other. An individual is thus conceptually tied to its identity with itself in a manner in which it is not with other relations.

A similar point is made by Castañeda in rejecting the suggestion that an individual might be individuated by instantiating the property of individuality:

> "Instantiation, which is an external connection between a property and what instances it, presupposes that a full-fledged entity fit to be the subject is already constituted. If such a subject cannot be an individual aside from instantiating the property of being an individual, this would make the account of individuation in terms of this property circular and useless" (*op. cit.*, p. 137).

Hence we are not suggesting that self-identity confers individuality, in the sense of acting as an individuator. All we are saying is that it is an essential feature of individuals (and here, we feel, Barcan Marcus got it right; *op. cit.*) whose denial gives us a way of formally representing the idea of non-individuals.

The nature of this formal representation has been presented elsewhere (see Krause 1992, 1996); here we shall only outline the central idea, before relating it to the issue of vagueness.

[28]Engels referred to '$a = a$' as 'abstract identity' and argued that it is 'inapplicable' in nature where change leads to an object, a plant say, being both identical with itself and yet distinct from itself at every moment (Engels 1940, pp. 214-216). Thus he observed that "Abstract identity, like all metaphysical categories, suffices for *everyday* use, where small dimensions or brief periods of time are in question; the limits within which it is usable differ in almost every case and are determined by the nature of the object" (*ibid.*, p. 215). We are grateful to John Divers for drawing our attention to this passage.

[29]Again quoted by Auyang (*op. cit.*).

7 Schrödinger Logics and Quasi-sets

In logic and mathematics the concept of identity is introduced in different and non-equivalent ways, depending on the language employed. In first-order languages, it is common to take a binary predicate as primitive, satisfying the reflexive law and the substitutivity principle (see Mendelson 1987).[30] The standard semantics then interprets that predicate in the diagonal of the considered domain; in other words, it is intended that the chosen predicate represents identity in the domain, although such a diagonal cannot be completely characterised by first-order languages. In higher-order languages, the predicate '=' can be defined ('ambiguously', in the sense of Russell) as a predicate of type $\langle k, k \rangle$ by the so called Leibniz' Law, where k is any type whatever.[31] Intuitively, this definition says that 'two' objects are identical (that is, are the very same object) iff they share all the same properties, that is, iff they are indistinguishable.

However, one can build logico-mathematical systems in which self-identity and (in)distinguishability are separated; that is, these concepts are not collapsed into one another. In particular, forms of logic –called Schrödinger logics– have been introduced for which '$a = a$' cannot be inferred for every a (da Costa and Krause 1994, 1997). These are many-sorted logics in that for a certain category of objects the expression $x = y$ simply is not a well-formed formula (and likewise for the negation $x \neq y$). For all other entities classical logic is maintained.[32] Correspondingly, there are set-theoretic considerations to deal with. Let us return to Lowe's characterisation of individuality in terms of countability. The question arises: How are we to understand a 'countable plurality' of individuals about which we cannot deny that it is indeterminate whether they are identical to themselves or not? In particular, in what sense can such a plurality be said to be countable?

Countability is precisely what is problematic about quantum entities. Teller, for example, has emphasised that 'quanta' cannot be counted but only *aggregated*, and invokes the well known analogy of money in a bank account previously deployed by Hesse and Schrödinger. This offers a further perspective on the way in which quantum particles might be regarded as non-individuals.[33]

How might we represent this distinction between a 'countable plurality' and an aggregate? One way is to note that it corresponds to that which holds between ordinality and cardinality respectively. Hence an 'aggregate' of quanta would have the latter but not the former, since we can say on experimental and theoretical grounds that there are, say, seven electrons in an atom, but we cannot count them, in the sense of putting them in a series and establishing

[30] In some first-order languages, as in standard set theory, the relation of identity can be defined, but we do not intend to pursue this here.

[31] See the definition of identity in Whitehead and Russell (1950).

[32] This is a characteristic of these systems, but in principle it is possible to define Schrödinger logics associated with non-classical logics. In da Costa and Krause (1997), a system is presented which encompasses modal operators. Other logics could be adequately used instead.

[33] Immediately after giving the pounds in the bank analogy Hesse states that "... it is meaningless to speak of the self-identity of electrons ..." (1970, p. 50).

an ordering.[34] However the distinction cannot be captured within the standard set-theoretic framework: If sets are understood as "... collections into a whole of *definite*, *distinct* objects of our intuition or of our thought" (cf. Cantor's 'definition' of the concept of set in Cantor, 1955, p. 85, our emphasis.) then from this perspective a collection or aggregate of quanta cannot be regarded as a set (as characterised, say, by Zermelo-Fraenkel axioms).[35] Quasi-sets have been proposed as a way of formally representing such aggregates (Krause and French 1995; French and Krause 1999) and, correspondingly, of providing a semantics for Schrödinger logics (da Costa and Krause 1997).[36]

One form of quasi-set theory –denoted by $\mathfrak{Q}$ (see Krause 1992, 1996)– allows the presence of two kinds of Urelemente: the m-atoms, whose intended interpretation are the quanta, and the M-atoms, which stand for macroscopic objects and to which classical logic is taken to apply. Quasi-sets are the collections obtained by applying ZFU-like (Zermelo-Fraenkel plus *Urelemente*) axioms to a basic domain composed of m-atoms, M-atoms and aggregates of them. The theory still admits a primitive concept of quasi-cardinal which intuitively stands for the 'quantity' of objects in a collection. The main idea is that the quasi-cardinal of a quasi-set cannot be associated (in the sense of this association being something described in the 'classical' part of $\mathfrak{Q}$) to a particular ordinal due to the (absolute) indistinguishability of the m-atoms, and this is the motive for taking this concept as primitive. This point notwithstanding, it is possible to define a translation from the language of ZFU into the language of $\mathfrak{Q}$ in such a way so that there is a 'copy' of ZFU in $\mathfrak{Q}$ (the 'classical' part of $\mathfrak{Q}$). In this copy, all the usual mathematical concepts can be defined, and the 'sets' (in reality, the '$\mathfrak{Q}$-sets') turn out to be those quasi-sets whose transitive closure (this concept is like the usual one) does not contain m-atoms.

In $\mathfrak{Q}$ there may exist quasi-sets whose elements are only m-atoms, called

[34]Expressions of philosophical disquiet on this point, to the effect that ordinality 'must' come before cardinality are typically grounded in certain metaphysical presuppositions regarding the concept of number. Thus if one is in thrall to the Kantian reduction of this concept to that of units of time, then one would indeed insist on the priority of ordinality over cardinality: "I do not find myself able to frame a distinct *conception of number* without *some* reference to the thought of *time*. ... I cannot fancy myself as *counting* any set of things without at first *ordering* them and treating them as successive' (Hamilton 1953, p. 125).

[35]Perhaps another way of approaching the deficiencies of standard set theory in this context is to note that it obscures a distinction between ways of 'gathering together' under a concept or rule, or as Cantor put it, different 'modes of conceiving a totality', which is reflected in that between an itemisation of elements and a statement of conditions for membership (Barcan Marcus 1993, p. 93). In mathematics, of course, the distinction doesn't arise but in physics we may be able to state membership conditions for an aggregate of quanta, without being able to itemise them. The distinction is striking in one of the examples used by Barcan Marcus where Suppes takes { Roosevelt, Parker } to mean "the set consisting of the two major candidates in the 1904 American presidential election". We can refer to the 'set', or rather, aggregate, consisting of the two electrons in the helium atom, without, however, being able to describe it by writing down the names of the members.

[36]Quasi-sets were proposed independently of Dalla Chiara and Toraldo di Francia's 'quasets', which also distinguish between cardinality and ordinality. The difference is that quasets are elaborated in terms of what are still regarded as individuals, where it may be unclear whether the individual counts as a member of the relevant quaset or not; for a comparison between both theories, see Dalla Chiara, Giuntini and Krause (1998).

'pure' quasi-sets and the axiomatics of the theory provides the grounds for saying that nothing in the theory can distinguish the elements of x from one another, for certain pure quasi-sets. Within the theory the idea that there is more than one entity in x is expressed by an axiom which states that the quasi-cardinal of the power quasi-set of x (the concept of subquasi-set is like that of standard set theory) has quasi-cardinal $2^{qc(x)}$, where $qc(x)$ is the *quasi-cardinal* of x (which is a cardinal obtained in the 'copy' of ZFU just mentioned). Now, what exactly does this mean? Our answer is as follows.

Consider the three protons and the four neutrons in the nucleus of a 7^{Li} atom. As we have indicated, nothing in the quantum mechanical description of the world distinguishes the three protons, say. Even so, physicists reason as if there were three protons there, and the physics does not work otherwise. In this way, if we regard these protons as forming a quasi-set, its quasi-cardinal is 3, and there is no contradiction in saying that all happens as whether there are also 3 subquasi-sets with 2 elements, despite we can't distinguish among then, and so on. So, it is reasonable to postulate that the quasi-cardinal of the power quasi-set of x is $2^{qc(x)}$. Whether we can distinguish among these subquasi-sets is a matter which does not concern logic.

In other words, we may consistently (with the axiomatics of $\mathfrak{Q}$) reason as if there are three entities in our quasi-set x, but x must be regarded as a collection for which it is not possible to discern its elements as individuals. The ground of such reasoning has been delineated by Dalla Chiara and Toraldo di Francia as partly theoretical and partly experimental. Speaking of electrons instead protons, they note that in the case of the helium atom we can say that there are two electrons because, *theoretically*, the appropriate wave function depends on six coordinates and thus "... we can therefore say that the wave function has the same degrees of freedom as a system of two classical particles" (*op. cit.*, p. 268).[37] Dalla Chiara and Toraldo di Francia also note that, "Experimentally, we can ionize the atom (by bombardment or other means) and extract two separate electrons ..." (*ibid.*). Of course, as we noted above, in our outline of the debate between Lowe and Hawley, the electrons can be counted as two only at the moment of measurement; as soon as they interact with other electrons (in the measurement apparatus, for example) they enter into entangled states once more. It is on this basis that one can assert that there are two electrons in the helium atom or six in the 2p level of the sodium atom or (by similar considerations) three protons in the nucleus of a 7^{Li} atom (and it may be contended that the 'theoretical' ground for reasoning in this way also depends on these experimental considerations, together with the legacy of classical metaphysics). On this basis one can state an axiom of 'weak extensionality' in $\mathfrak{Q}$, which says that those quasi-sets that have the same quantity of elements of the same sort (in the sense that they belong to the same equivalence class of indistinguishable objects) are indistinguishable.[38]

[37]This might be regarded as the legacy of the 'Schrödinger' formulation which "gets off on the wrong foot" by initially assigning particle labels and then permuting them before extracting combinations of appropriate symmetry (Post 1963).

[38]In an earlier version of the theory it was postulated that only similar quasi-sets with the

8 Vagueness and Self-Identity

With this formal framework in mind, let us return to the metaphysical issues. There is a commonly held 'intuition' that something which does not possess a determinate self-identity simply cannot be regarded as an *object* of any sort (Lowe 1994b, p. 533). This intuition, or something like it, was expressed by Quine in his slogan: 'No entity without identity'. What underlies it, of course, is the feeling that dropping identity would be too disruptive of our logic. As part of an argument against the view being explored here, it is entirely question begging of course. Furthermore, we may acknowledge with Barcan Marcus that,

> "... all terms may 'refer' to objects, but ... not all objects are things, where a thing is at least that about which it is appropriate to assert the identity relation. ... If one wishes, one could say that object-reference (in terms of quantification) is a wider notion than thing-reference, the latter being also bound up with identity and perhaps with other restrictions as well, such as spatiotemporal location" (*op. cit.*, p. 25).

Thus for her, the appropriate slogan is 'No identity without entity' (*op. cit.*).[39]

Sloganeering aside, there is further work to be done. With quanta, as we have seen, we have situations where the sentence "there is exactly one F" is true yet the description term "the x which is F" fails to have a denotatum. Perhaps we have to abandon reference in anything other than the broad 'object' sense indicated by Barcan Marcus above. In that case we need to closely consider the grounds for asserting 'there is exactly one F'. Such grounds are experimental and theoretical, as we have just indicated. It is the number of quanta in an 'aggregate' –that is, the cardinality of the appropriate quasi-set– that is typically measured in a scattering experiment, for example. Auyang refers to such numbers as 'partial manifestations' of the properties of a quantum system in experiments (*op. cit.*, p. 160).[40] Dalla Chiara and Toraldo di Francia write of a kind of 'mock' individuality of quantum objects arising out of these experimental manifestations. As we noted above, it is on this basis that we can refer to the electron as b but the label is only a 'mock' form as until the measurement inetraction occurs, the particle is still in an entangled state. Within the context of measurement, of course, we have little choice but to conceptualise within a classical framework (classical in both senses!).[41] As a first approximation we

same quasi-cardinality were indistinguishable quasi-sets. See Krause (1996).

[39] The difference is tied up with a different understanding of quantification and the way this functions as a guide to ontology. For Barcan Marcus, "... if we want to *discover* which objects a language or theory takes to be individuals, we look to see which objects are such that they can meaningfully enter into the identity relation" (*op. cit.*).

[40] "To say the field is in a state $|n(k_1), n(k_2), \ldots\rangle$ is not to say that it is composed of $n(k_1)$ quanta in mode k_1 and so on but rather $n(k_1)$ quanta show up in an appropriate measurement." (Auyang 1995, p. 159).

[41] Dalla Chiara and Toraldo di Francia refer to this as 'objectuation' and suggest that it is a necessary prerequisite to all discursive activity (1993).

conceive of the particles as individuals, but as we move into situations of entanglement we find that that approximation breaks down in one of the two ways explored here.

There is an analogy at this point with Quine's discussion of the problem of the individuation of attributes and propositions, which is the context for the above slogan (1969, p. 23). Quine argues that "... the positing of first objects makes no sense except as keyed to identity" (*ibid.*). If 'first objects' are read as classical particles, then where Quine writes of the inculcated 'patterns of thing talk' enabling us to talk of attitudes and propositions in 'partial grammatical analogy', without an accompanying standard of identity, we might so talk of quantum particles, where the 'partial analogy' is here metaphysical.[42]

The consequences of such a move are clear:

> "What might properly count against countenancing such half-entities, inaccessible to identity, is a certain disruption of logic. For, if we are to tolerate the half-entities without abdication of philosophical responsibility, we must adjust the logic of our conceptual scheme to receive them, and then weigh any resulting complexity against the benefits of the half-entities in connection with propositional and attributary attitudes and elsewhere"(*ibid.*).

It is of course precisely such an adjustment of the logic of our conceptual scheme that Schrödinger logic and quasi-set theory offer and within the latter the M-atoms may be regarded as playing the role of Quine's 'first objects'. The benefits of our scheme accrue with regard to the metaphysical component of our conceptual scheme in accommodating quanta.

Coming back at last to the arguments concerning vagueness, some have insisted that it is a necessary condition of countability that

> "... the items to be counted should possess determinate identity conditions, since each should be counted just once and this presupposes that it be determinately distinct from any other item that is to be included in the count" (Lowe 1994b, p. 536).

Thus, on this view, if a plurality is countable, the entities of which it is composed must possess self-identity. Returning to the electron example, the fact that there is a determinate number of electrons presupposes, it is claimed, that each of them is one and therefore one and the same with itself and therefore an individual (Lowe 1994a). However, the notion of 'countability' that is so important here is ambiguous and, dare we say it, vague. In particular we need to distinguish cardinality from ordinality and this is precisely achieved in quasi-set theory, as we have indicated[43]

The elements of certain quasi-sets are countable, *in the sense of possessing (quasi) cardinality but not in the sense of possessing ordinality.* Since it is

[42] Referring to attributes and propositions, he asks "Why not accept them thus, as twilight half-entities to which the identity concept is not to apply?" (*ibid.*).

[43] As we have noted, Lowe now accepts that countability does not imply determinate identity.

strictly the latter, rather than the former, that is involved in the above notion of 'countable', the line of reasoning which takes us from 'determinate number ...' to individuality collapses. The upshot then, is that we can have a determinate number of quantum objects in a given state without these objects possessing definite identity conditions. And it is because of this lack of self-identity that the objects can be described as vague, in perhaps the most fundamental sense one can imagine.

9 Conclusion

It is our firm belief that philosophers in general and metaphysicians in particular should take seriously the accounts which science presents about 'the world', however understood. By relating such accounts to areas of metaphysics, for example, illumination may be obtained for issues in both domains. What we have attempted to do here is to extend Lowe's analysis of vagueness in quantum mechanics and elaborate two ways in which ontic vagueness can arise in this context. If the particles are regarded as individuals, then such vagueness is a result of the existence of non-supervenient relations, which effectively cast a 'veil' over the set of entities. If, on the other hand, it is insisted that individuality must be given up in this domain, then the particles are vague in an even more fundamental metaphysical sense. Yet still, we would argue, they can be considered as objects and quasi-set theory provides an appropriate formal representation of collections of such objects.

The source of this metaphysical vagueness in all cases is the 'entangled' states which quantum particles enter into. It is such states which lie at the heart of the quantum mystery and which force all the attempts at metaphysical understanding, vague or otherwise. We believe that such understanding might be achieved by tackling the metaphysics head on, as it were, and considering these states in terms of basic metaphysical categories such as that of individuality. That more than one such metaphysical package might be the result will not satisfy the naive realist; nevertheless, that such packages can be elaborated and related to 'standard' philosophical discussions of issues such as vagueness is, we feel, a significant result.

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