

Quantum Mechanics and Language

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The language of quantum physics

- Schrödinger, "What is an elementary particle?"
 - "[w]e have taken over from previous theory [classical mechanics]
 - the idea of a particle and all technical language concerning it.
 - This idea is inadequate. It constantly drives our mind to ask for
 - information which has obviously no significance. Its imaginative
 - structure exhibits features which are alien to the real particle.
 - (...) The particle (...) is not an identifiable individual. (...)
 - It is not at all easy to realize this lack of individuality and to find words for it".

- Yuri Manin 1976 Problems of Present Day Mathematics

- "In accordance with Hilbert's prophecy, we are living in Cantor's Paradise. So we are bound to be tempted.

- (...)

- (b) We should consider the possibilities of developing a totally new language to speak about infinity. Classical critics of Cantor (Brouwer et al.) argued that, say, the general choice axiom is an illicit extrapolation of the finite case.

- (...)

- I would like to point out that this is rather an extrapolation of common-place physics, where we can distinguish things, count them, put them in some order, etc. New quantum physics has shown us models of entities with quite different behavior.

- Even 'sets' of photons in a looking-glass box, or of electrons in a nickel piece are much less Cantorian than the 'set' of grains of sand. In general, a highly probabilistic 'physical infinity' looks considerably more complicated and interesting than a plain infinity of 'things'.

- (..)

- The twentieth century return to Middle Age scholastics taught us a lot about formalisms. Probably it is time to look outside again. Meaning is what really matters".

Non Relativistic QM

(A structure for QM, according to M.L. Dalla Chiara and G.Toraldo di Francia)

- **MC = $\langle F, M \rangle$**
- **F = mathematical formalism**
- **M = class of models**
- **M = $\langle S, A, Q, Q, \dots, Q, f \rangle$**
- **S = *set* of physical systems**
- **A = a model of standard functional analysis**
- **Q_i = physical variables**
- **For every $s \in S$, $f(s) = H$ (an Hilbert space in A)**
- **For every s and Q , $f(Q) =$ self-adjoint operator in $f(s)$**
- **Etc.**

Semantics

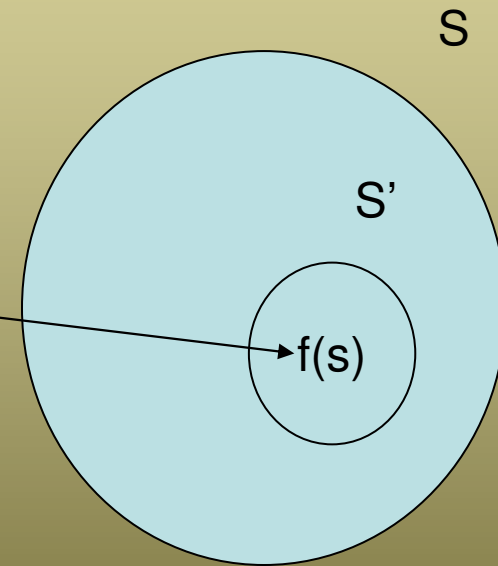
- L (the diagram language)
- **Names for the elements of S**
- **Predicates (unary) whose extension are subsets of S .**
- **$M = \langle S, A, Q, Q, \dots, Q, f \rangle$**
- For every name (individual constant) a of the language of QM , and for every property P , the schema attributes a subset $S' \subseteq S$ and an individual $s \in S$ such that $P(a)$ is true according to this interpretation, iff $f(s)$ is in S' .
- $M \models P(s)$, iff $f(s) \in S'$.

The ('classical') semantic schema

L

s (name –individual
constant)

P (unary predicate)



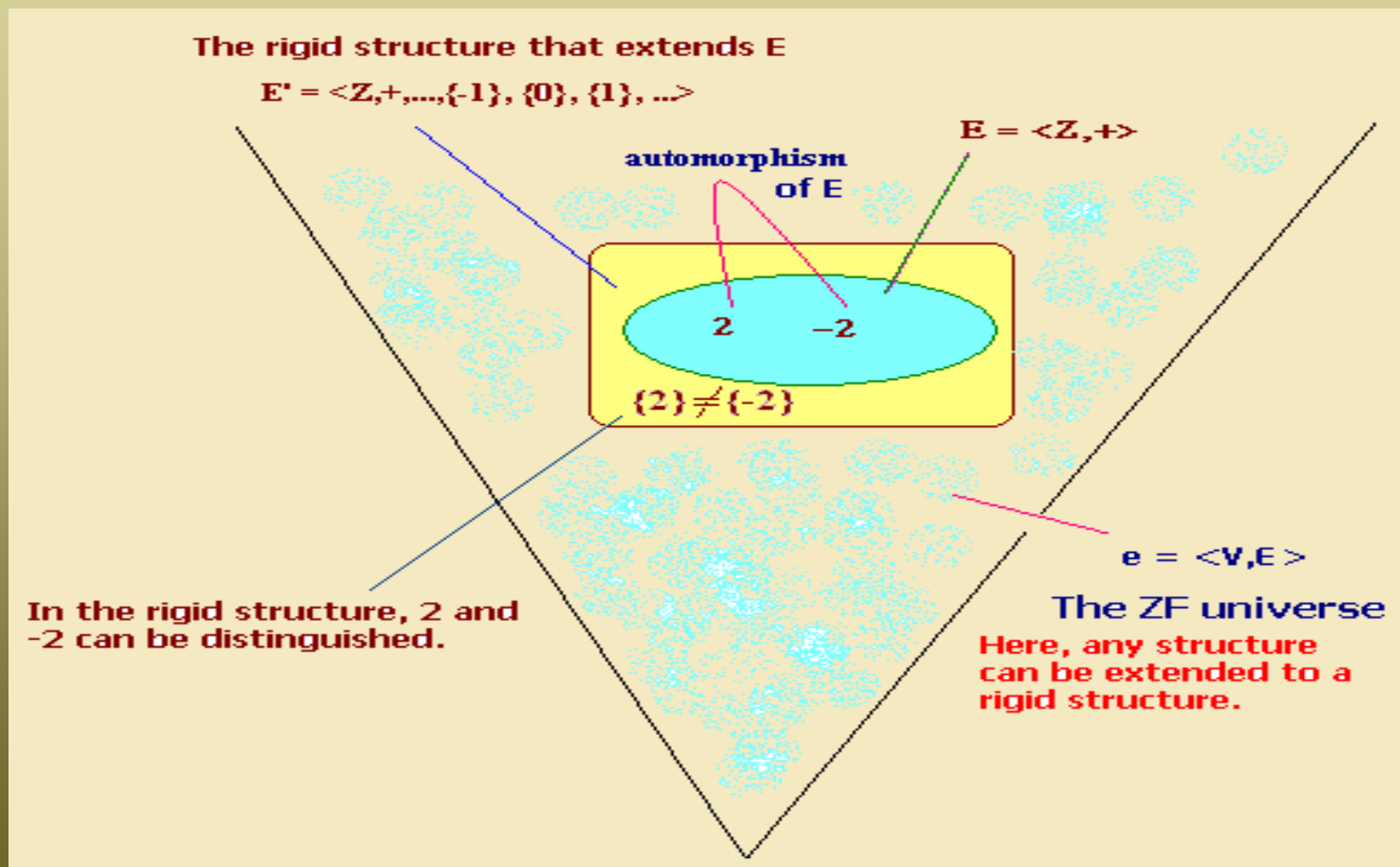
Performed in ZF

$$M \models P(s) \text{ iff } f(s) \in S'$$

Indiscernibility in a structure

- A **rigid structure**: its only automorphism is the identity function
- Ex: $A = \langle \mathbb{Z}, + \rangle$ (in ZF) is not rigid
- The automorphisms of A : $i(x) = x$ and $f(x) = -x$
- From inside of A , 2 and -2 are indiscernible
- But they are not indiscernible from the *outside* of A .
- That is, in the 'whole ZF structure', $E = \langle V, \in \rangle$
- We can extend A to a rigid structure $A = \langle \mathbb{Z}, +, \{0\}, \{1\}, \{-1\}, \dots \rangle$, where any element of \mathbb{Z} is an individual.
- **Theorem (ZF)**: Every structure can be extended to a rigid structure.
- This is true also for $\mathbf{M} = \langle \mathbf{S}, \mathbf{A}, \mathbf{Q}, \mathbf{Q}, \dots, \mathbf{Q}, \mathbf{f} \rangle$

In ZF,
indiscernibility is always relative to a certain structure.



- $M = \langle S, A, Q_0, \dots, Q_n, f \rangle$
- Is a *structure* in ZF
- An adequate structure for QM should not be able to be extended to a rigid one.
- It's necessary to change the meta-mathematics.
- For a mathematics encompassing collections of indistinguishable objects.
- This is quasi-set theory.