Skolem and van Fraassen's Paradox

A look at the "models" of a scientific theory

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Motivation-1

van Fraassen:

- "If two particles are of the same kind and have the same state of motion, nothing in the quantum-mechanical description distinguishes them. Yet this is possible."
- Question: If they cannot be distinguished *inside* the theory, (perhaps) they can be distinguished only *from the outside* of the theory.

Motivation-2

van Fraassen's modal interpretation of QM

- It introduced the distinction between *value-attributing properties* and *event-attributing properties,* or between *value states* and *dymanic states.*
- Propositions about *states* are described *within* the scope of QM.
- Propositions about *events* lie (although "not completely") *outside* the scope of (the formalism of) QM.

Skolem's "relativity" of set-theoretic notions:

- Skolem highlighted the importance of paying attention to the possibility of looking at concepts *inside* a model and *from ouside* that model.
- For there are concepts which look the same in all models and concepts that do not.
- *Cardinal* is a concept that depends on the model: there are distinct models of set theory in some of which **R** is denumerable and in some it is not (Skolem's paradox).
- *Ordinal* is an absolute concept – it *looks the same* in all models.
- van Frrassen used (even if implicitly) a similar strategy in the empirical sciences.
- And we think that we should pay attention to the importance of that possibility also in the empirical sciences.

A first philosophical point

- Following the semantic approach, a scientific theory is identified *via* a class of models.
- Models are usually taken as set-theoretic structures.
- a) Is there something in science similar to the distinction between relative and absolute concepts in set theory?
- b) Are there concepts or notions that are the same in all models?
- c) Are there notions that change from model to model?
- d) Which would be good examples?

Philosophical Implications

- More:) The models of the relevant theories *are not* first-order structures, like $A = \langle D,I \rangle$ (where D is a non empty set and I is the interpretation function), but they are *higher-order structures*.
- Thus, it is doubtful in what sense the techniques of (first order) model theory apply to these models.
- Further, the “construction” of these models presupposes some mathematics
- In fact it presupposes a particular set theory
- However, there are several non-equivalent set theories.
- In which way does the choice of a particular set theory (or a particular “model” of a set theory) matter for philosophical discussions?

Some examples

1) Unbounded operators are essential in quantum physics.

- But in Solovay's set theory, all linear operators on a Hilbert space are bounded.
- 2) The standard quantum formalism depends essentially on the orthornormal basis of eingenvectors of some operators.
- In some of Hans Läuchli models of set theory, we have: (1) vector spaces with *no basis*; (2) vector spaces with *basis of different cardinalities*.
- Can we formalize QM using these set theories (models)?
- If so, at what price?
- The metamathematics employed may impose limitations on what can be said about the *theories* under consideration as well as about their *models*.
- These metamathematical considerations should be examined much more carefully and thoroughly.

A second philosophical point

The situation in current particle physics

- There is no unification of QED, QCD, and Gravitation.
- Strictly speaking, there are no *theories* (formulated from clear first principles, with a list of axioms, etc.).
- Arthur Jafee: "Yet in spite of these great successes [high precision], we do not know if the equations of QED make mathematical sense. […] Most physicists today believe that the equations [we can read: the axioms] of QED […] are inconsistent."
- Are we then in a different paradigm than the one expressed by A.S. Wightman when he said:
- "A great physical theory is not mature until it has been put in a precise mathematical form"?
- (Edward Witten said that no one is actually apt to write the basis of a TOE – Theory of Everything.)

A consequence

- Having no “theories” in strict sense, how can we speak of models?
- Models of what?
- Strictly speaking, there are no “models” *tout court* here either.
- For models are models of “something” (e.g. models of the theory’s axioms).
- The concept of model must be adjusted to accommodate these cases.

An additional point

- These “theories” (QED, QCD, Standard Model) do not have iconic models, “representational models”, or anything that “stands for or represents *reality*”.
- At this level, it seems that there is no reality outside the “theories” (theories understood as informal mathematical devices plus some rules of thumb to interpret the formalism in some physically sensitive way).
- No one could imagine quarks without (or “outside”) quantum mechanics.
- In order to conceptualize quarks, we need to invoke the relevant mathematical theories.
- How could we elaborate a model for quarks without considering the relevant mathematical devices and structures?
- Could we consider quarks as little *colored* balls with *flavors*?
- Quarks have no “representation” outside the context of the suitable mathematical theories.

- Thus, it is doubtful if we can make "abstractions" as we do when we consider a frictionless pendulum in other parts of physics.
- After all, what are we abstracting from?
- In a certain sense, the entities *are* their mathematical descriptions, since we are unable to look "outside" the theory.
- These *entities* are posited by the theories as certain mathematical structures: they are, in a sense, mathematical structures.
- What about their counterpart in reality?
- Considering such an "external" question raises some metaphysical issues (*pace*, Carnap).

- An example: the problem of *identical particles*:
- According to van Fraassen, this is one of the three main issues in the philosophical foundations of QM.
- Bosons (e.g. in a BEC – *Bose-Einstein condensate*) may be indistinguishable in quite absolute terms.
- But bosons are "particles", even being certain excitations of the fields.
- The "basic ontology" is an ontology of fields.
- The underlying logic and mathematics of field theories are classical, thus all objects obey the traditional theory of identity (TTI).
  - Hence, these (fields and particles) objects are *individuals*.

- As individuals, identical particles:
- should bear names, labels;
- should be counted;
- should be seen as referents of singular terms (in standard semantics)
- should be re-identified (*genidentity*).
- A collection of identical particles should be viewed as a *set* in standard set theories (that is, as collections of *distinct* objects).
- Identical particles should always be distinguished from others, even of similar species or kind.
- The underlying logic (and mathematics), that is, the "logic of individuals", tells us a story…

- …which apparently does not agree at all with the story told by QM.
- For, according to quantum theory, identical particles should be such that:
- They cannot (in general) be distinguished.
- They cannot bear names (as rigid designators).
- Collections of indistinguishable objects *cannot be sets* in the usual sense.
- We cannot define a reasonable semantics (that is, a *standard,* or classical, semantics based on a standard set theory) for a quantum language based on indistinguishability.

Questions

- Are identical particles indistinguishable only *inside* the theory?
- Can they be distinguished from the *outside*?
- Or are they distinguished only *in mente Dei?*
- 1st option: identical particles have a *substratum*, a *quid,* that cannot be described by more basic properties.
- 2nd option: it is assumed that identical particles obey classical logic (in particular, the classical theory of identity), but that only "quantum properties" should be considered.
- Both options raise strong metaphysical assumptions.
- Are we reintroducing here something like Carnap's external questions?

The 2nd option

- This option (namely, that only "quantum *internal* properties" should be considered) introduces a kind of hidden variables: the assumption that there are properties which are not "quantum".
- How can bosons obey B-E statistics by preserving equal attributions of probabilities to all possibilities? Is this unimportant?
- Perhaps this leads us to a distinction similar to van Fraassen's *value-attributing properties* and *event-attributing properties.*
- But, in this case, this option does not entirely satisfy us.

Our claims

About the 1st philosophical point

(the mathematics used)

- If we keep the emphasis on the semantic approach, the discussion about the mathematics and the logic used to describe the models must be seriously taken into account.
- After all, a change in logic and in mathematics may change the meaning of some concepts (those we can call, following Skolem, *relative concepts*).
- We guess that we need to consider to what extent there is a distinction between relative and absolute concepts in scientific theories, similar to what happens in the foundations of set theory.

About the 2nd philosophical point

(no theories, no models)

- The semantic approach seems to be questioned.
- For the current situation in physics suggests that perhaps we will not have *theories* in the strict sense (with a language, primitive concepts, axioms etc.).
- We won't have even *models* (in the standard sense).
- Rather, we just have a cluster of mathematical heuristic devices that can be used simultaneously (and, in some cases, inconsistently) to attack a particular problem, and to solve it.
- Thus, to accommodate the situation in current science, the concept of *model* must be re-analyzed, as well as the semantic view of theories.

- **THAT'S ALL**

- B-E statistics for individuals

A	B	prob
a, b		1/3
	a, b	1/3
a	b	1/6
b	a	1/6