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Logic and Quantum Physics

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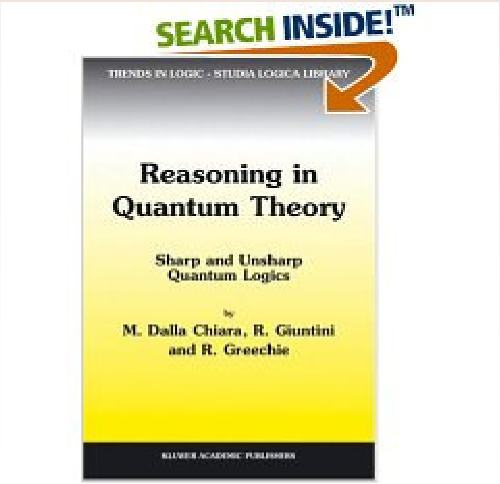
Quantum Logic

- 1936 Birkhoff and von Neumann
- *'quantum logics'*
- Piron, Jauch
- P. Mittelstaedt
- E. Beltrametti
- Etc.
- The study of certain lattices
- (the complete orthomodular lattice of closed subspaces of a Hilbert space)

- 2004 **Reasoning in Quantum Theory : Sharp and Unsharp Quantum Logics (Trends in Logic)** M. Dalla Chiara, R. Giuntini, R. Greechie, 320 pp., Springer; 2004.



Quantum Physics
vs.
Quantum Mechanics

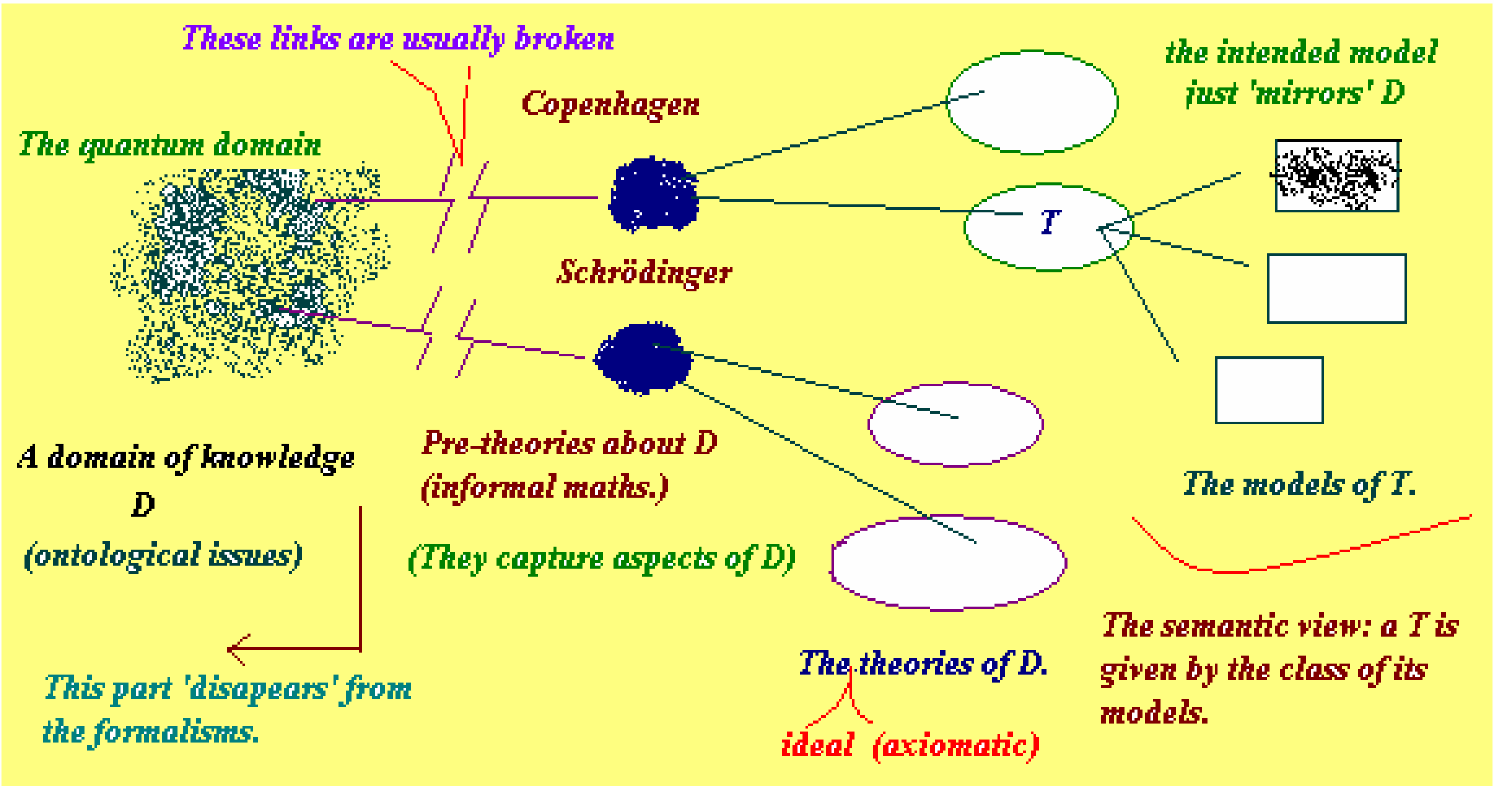
- *Quantum Physics* (QP) – general informal view of the ‘quantum world’
  - the physics of the small
  - at the ‘Planck’ scale (≈ 10^{-33} cm)
  - the realm of indiscernible, ineffable, objects
  - There are proposals of a ‘new ontology’

  Quantum Mechanics (QM) – the ‘(pre)theories’ about QP
  - Heinsenberg, Schrödinger, Bohm, Dirac, QFT, etc.
  - The “partial” *quantum (pre)theories* about D.
  - Using standard mathematics (and logic)
  - (Reichenbach, Février -- other logics)

Pluralism

(of views, of logics, of maths., of …)

The 'domain' of QP motivated the development of QM, but only to a certain point: the resulting formalism [truly, several of them] "was the outcome of a complicated conceptual process of trial and error and it is hardly an overstatement to say that it preceded its own interpretation, a development almost unique in the history of physical science." (M. Jammer, The Phil. of QM, p. 2).



The needs for a quantum logic

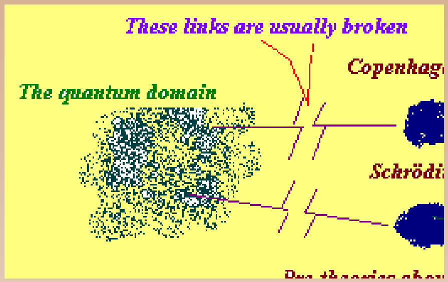
- Different interpretations (or quantum (proto)theories)
- Schrödinger: The ontology of *waves* (but they are also *objects* of some kind) – standard mathematics (and logic)
- Heinsenberg, Bohr: the *Copenhagen* view
  - They reject the objective reality of the quantum world
  - We only describe our experiences with macroscopic apparatuses.
  - "QM is a mathematical scheme that relates the results of observations in a statistical fashion." -- standard mathematics (and logic)
- Dirac, Bohm, etc.
- Carl von Weizsäcker

  Epistemological propositions: "The measurement of the observable P has value k"

  Ontological propositions: "The physical object x has the property P"
- Physical theories have some kind of 'objectivity', that may be ontological or simply experimental.

The ‘object’ in *quantum physics*

- (in the quantum domain)



- P. Mittelstaedt, P. Weingartner, P. Teller, M. Redhead,...
- *Quantum Ontology (how QM works)*
  - They are objects of predication: they have properties and can be classified in species (particle physics)
  - They can be aggregated in certain amounts (collections of quanta): BECs
  - There are sharp properties and vague objects

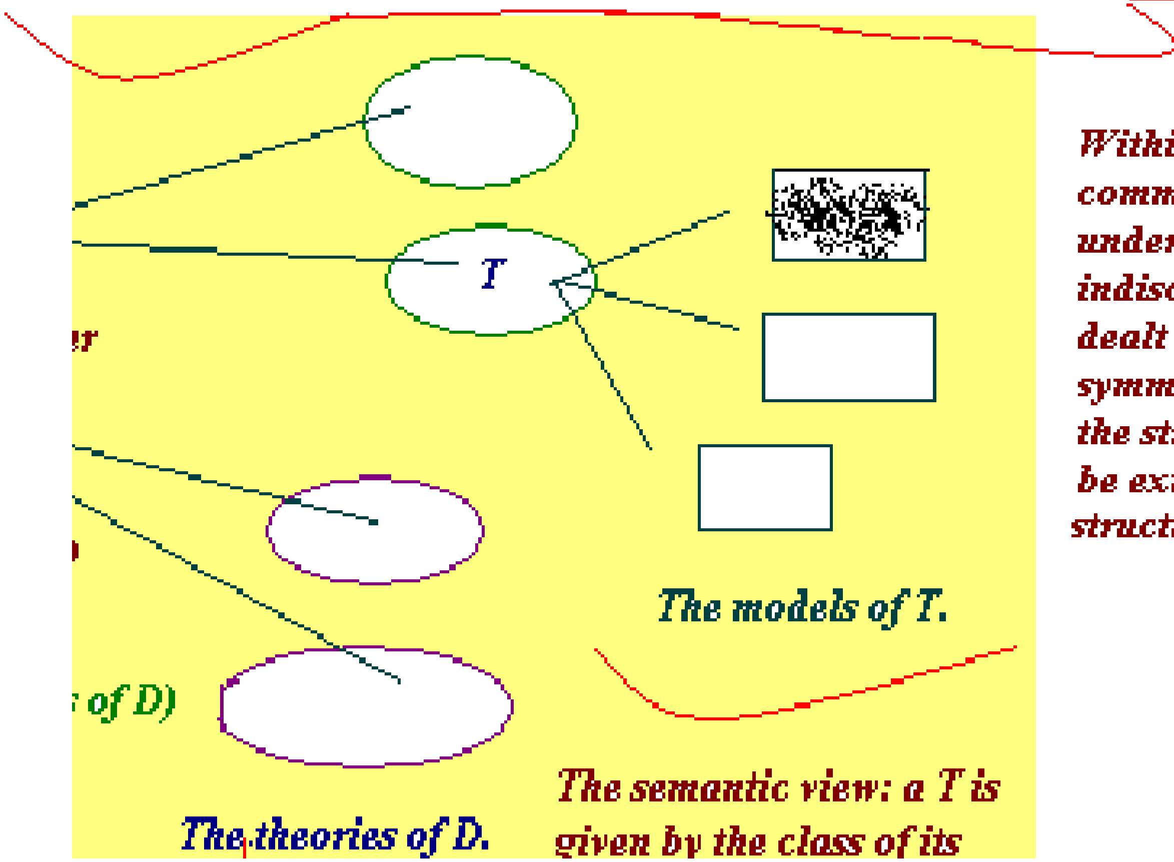
Quantum objects (cont.)
--are they supposed to exist?--

- They enter in entangled states
  - No distinction is possible
- There are incompatible observables
  - *e.g.,* position and momentum
- They may be absolutely indistinguishable:
  - Planck's derivation of the Black Body Radiation Law
  - B-E statistics
- They cannot be named, labeled, counted
  - Possible *names* are only 'mock' names
  - There are no *rigid designators* (*genidentity*)
- Individuals and non-individuals:
  - two possible metaphysics

In the formalisms, there is no 'semantic' counterpart for such (only) 'formal' entities.

The commitment to standard languages

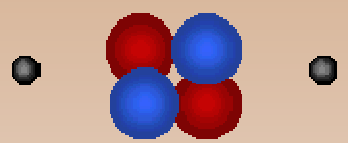
Here we work within a 'standard' set theory.



Within set theory, we are commited to obey its underlying principles: indiscernibility can be dealt with only via symmetry conditions, the structures can always be extended to a rigid structure, etc.

The 'classical' treatment

http://web.jjay.cuny.edu/~acarpi/NSC/3-atoms.htm



The Helium atom consists of 2 electrons and a nucleus made up of 2 protons and 2 neutrons.
Are there differences?

The Ground State

"We begin with the strongest effect, that of the electron Coulomb interaction with the nucleus. In that case we can write the [symmetric] Hamiltonian as:

$$H = \left(-\frac{\hbar^2}{2m}\nabla_1^2 - \frac{2e^2}{r_1}\right) + \left(-\frac{\hbar^2}{2m}\nabla_2^2 - \frac{2e^2}{r_2}\right) = 2H_0 \qquad (Z=2).$$

The subscripts 1 and 2 refer to the electrons 1 and 2."

http://tesla.phys.unm.edu/phy537/11/node4.html

- **QP (quantum physics) presupposes indiscernibility, while classical mathematics can deal with indiscernible objects only if restricted to a certain framework, or *structure*.**

How to treat them within classical maths?

Do the ends justify the methods?

In the Machiavellian QM, apparently this is the case.

- "The coordinates of the electrons are labeled 1 and 2 under the provisional assumption that the particles are in principle distinguishable. Of course, we know that this assumption is false but (…) with this assumption we can obtain the entire spectrum of the two-electron system."

- (E. Merzbacher, *Quantum Mechanics*, 2nd. Ed., John Wiley, 1970, pp. 442-3)

Agent Smith 1 Agent Smith 2

Who is Agent Smith?

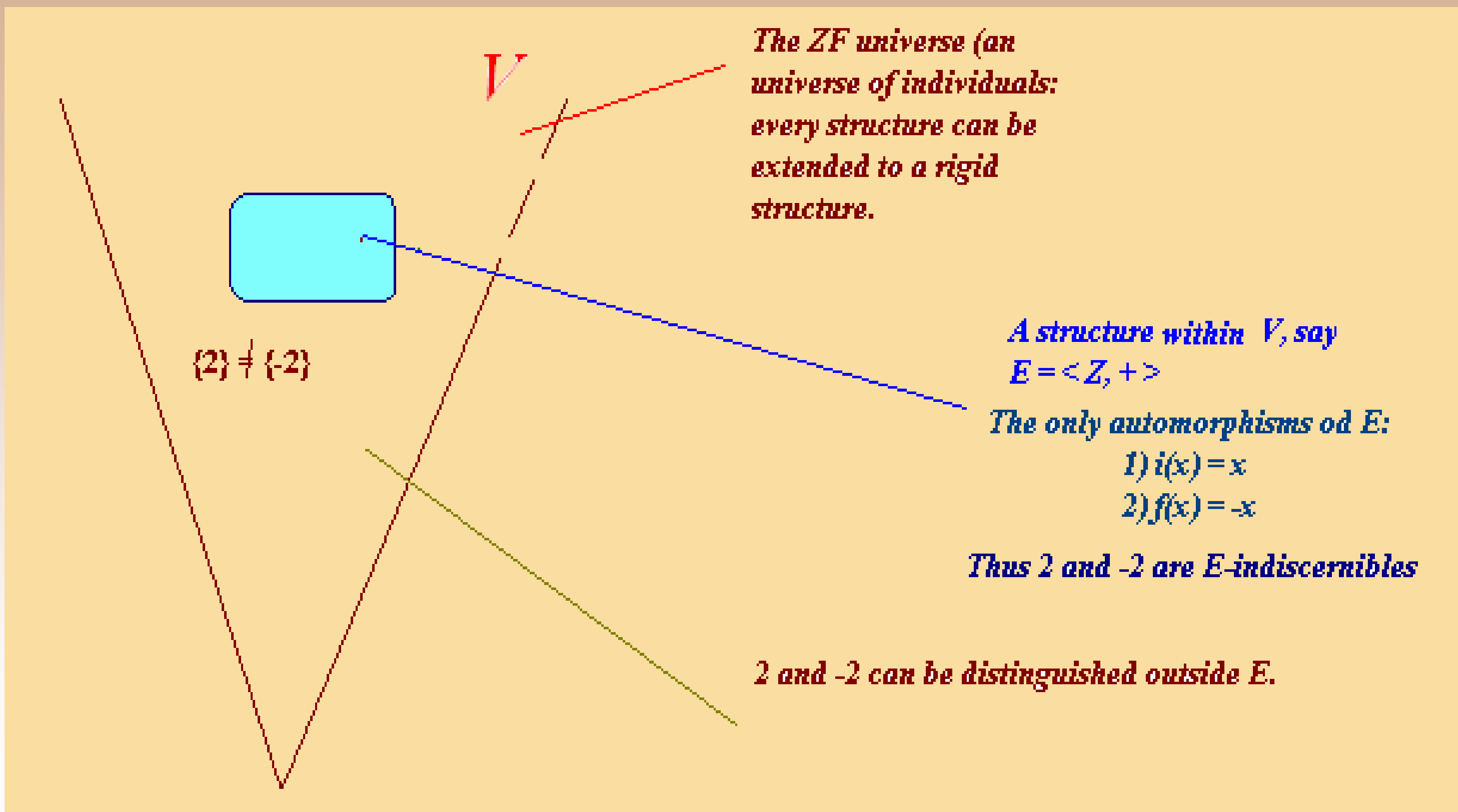
Does it matter?

How quantum mechanics deals with *them*

- We suppose that *they* are individuals (for instance, by ascribing labels to them).
- Symmetry Postulates are introduced:
  - Only symmetric (or anti-symmetric) functions or vectors have physical meaning
  - $\langle\psi|Q|\psi\rangle = \langle P\psi|Q|P\psi\rangle$, for all P and Q
- An object <u>a</u> described by $\psi(a)$, and an object <u>b</u> described by $\varphi(b)$ have the composed state given by:
  - $\Psi(a,b) = \psi(a)\,\varphi(b) \pm \psi(b)\,\varphi(a)$ (in a simple form)
- What is important? $|\Psi(a,b)|^2$ (Born's rule)
- The very *objects* are ruled out from quantum theories (mainly in QFT)
- But not from its *pragmatic interpretation* (just look at a book on 'Particle Physics')

‘Standard maths’:

the ZF well-founded universe (perhaps involving *Urelemente*)



Indiscernibility in standard maths.

- Within standard mathematics, the most we can do it to *mimic* indiscernibility inside a certain structure.
- An example with a first-order structure:
- Within $E = \langle Z, + \rangle$, 2 and -2 are indiscernible
    - The only automorphisms of *E* are f(x) = x and g(x) = -x
    - 2 and -2 are invariant under the automorphisms of E.
    - The *orbit* of n, orb(n) = {n, -n} = orb(-n)

- We can extend $E = \langle Z, + \rangle$ to a rigid structure e.g., ,
  A = < Z, + , {0}, {1}, {-1}, ...>, in which all elements of Z are individuals.

- *Theorem:* In ZF, every structure can be extended to a rigid structure (its only automorphism is the identity function).

    – Every 'ZF-object' is an *individual*.

- This theorem can be extended to higher-order structures
  (*via* Generalized Galois Theory)

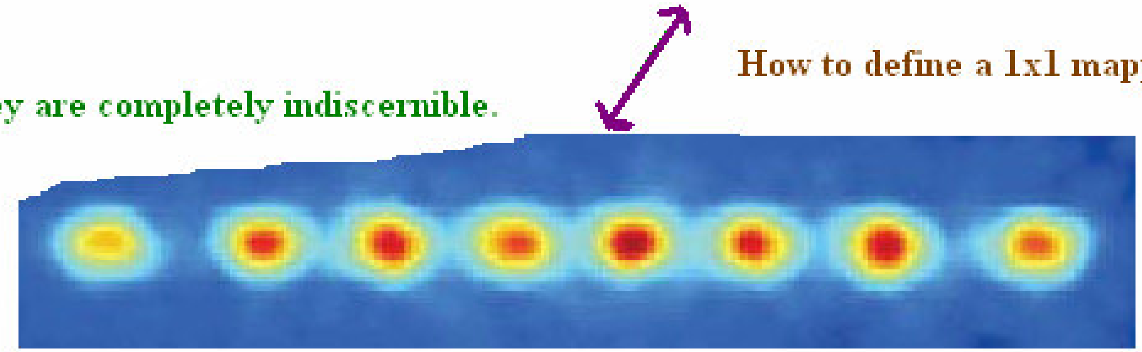
The empirical situation

Trapped Ions

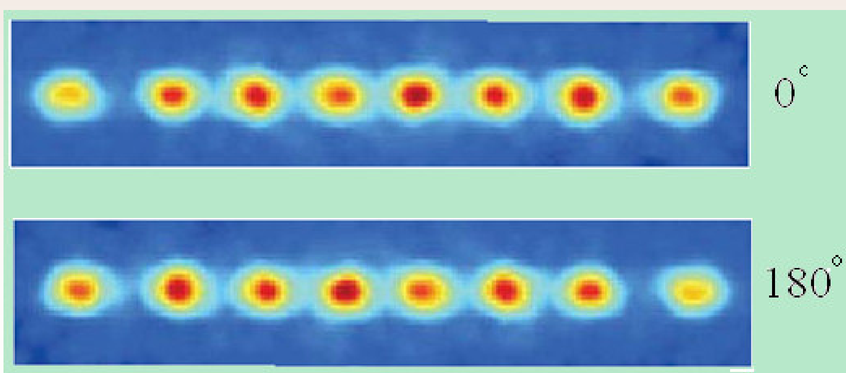
$8 = \{0,1,2,3,4,5,6,7\}$

How to define a 1x1 mapping?

They are completely indiscernible.

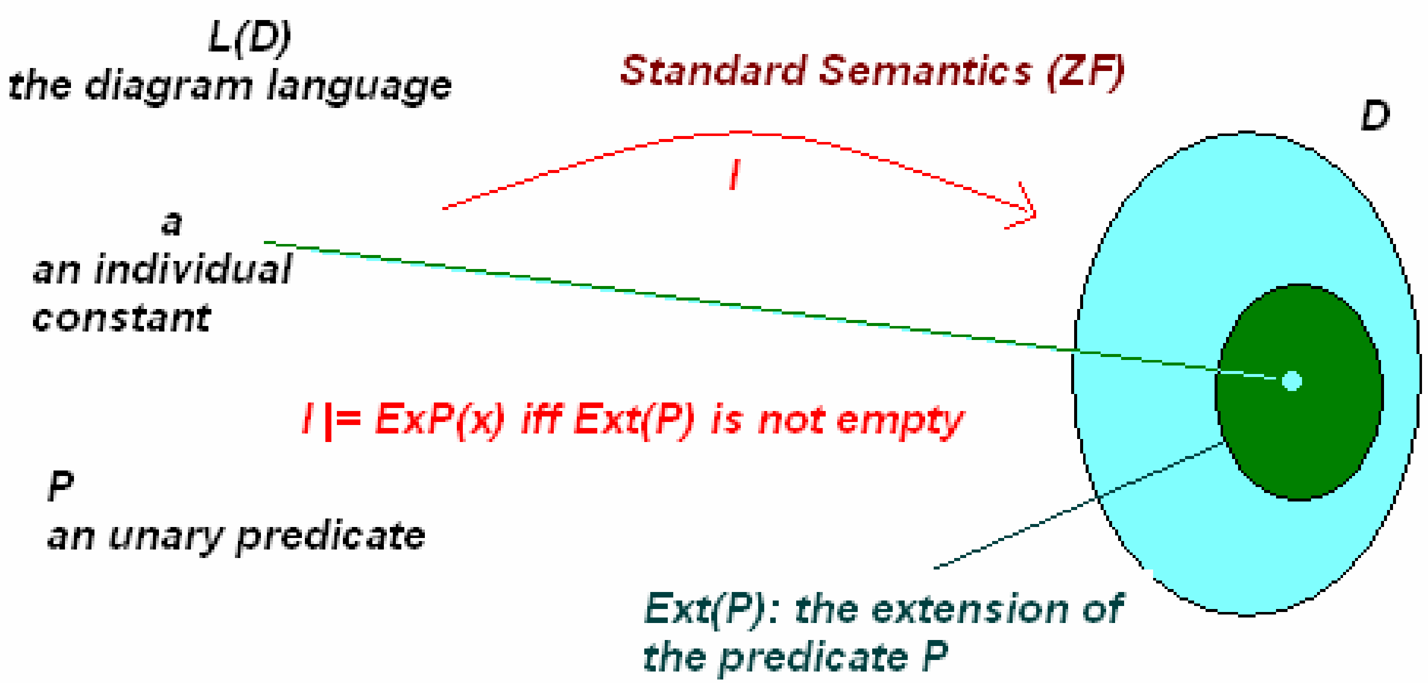


ENTANGLED: Eight calcium ions held together in a trap are in a special quantum condition known as a W entangled state, in which their properties are subtly correlated. *Sci. Am. March 2006.*



It is not the case that whatever permutation can be extended to an automorphism of the structure: these words simply have no meaning here!

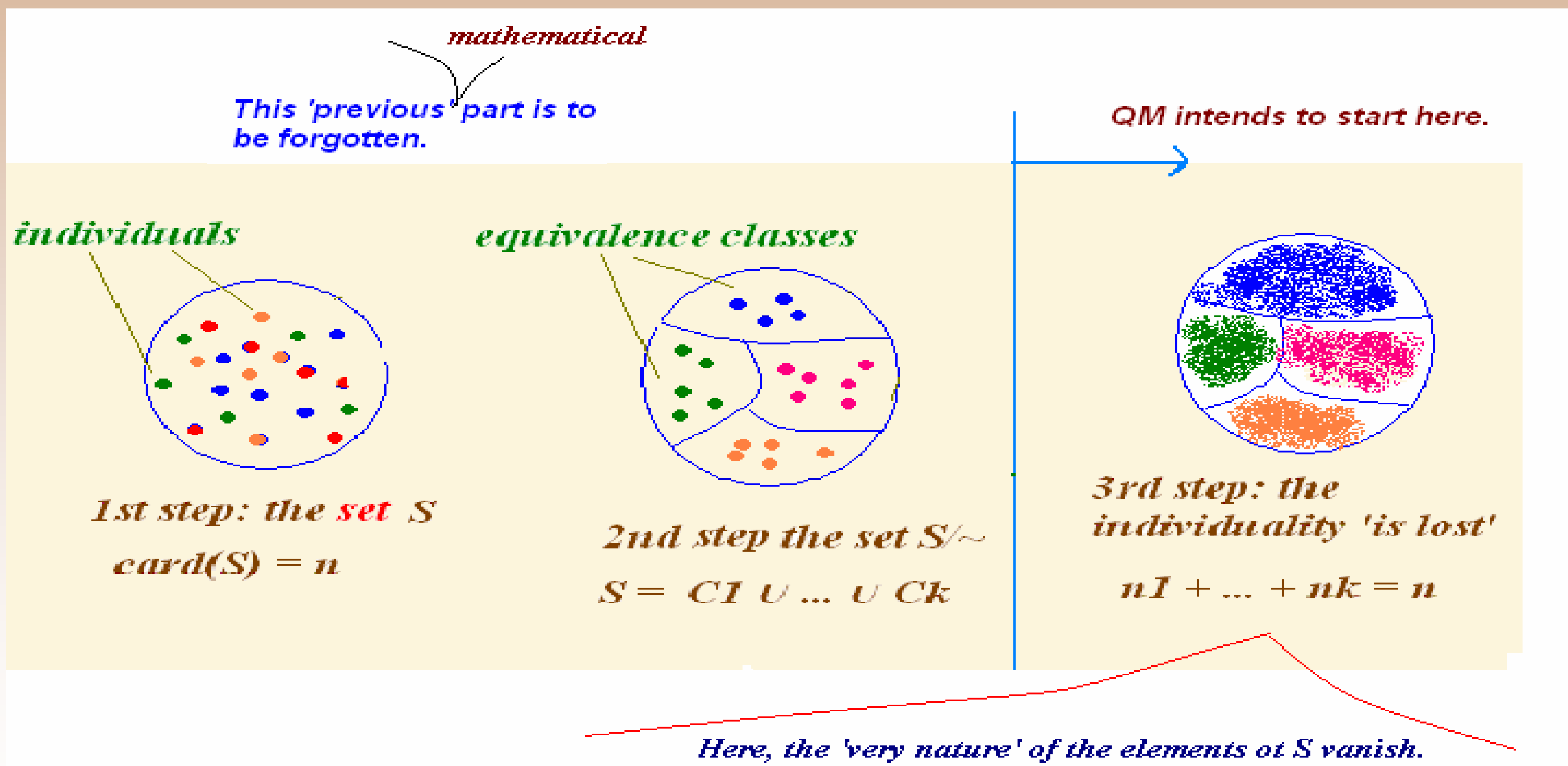
‘Classical’ semantics



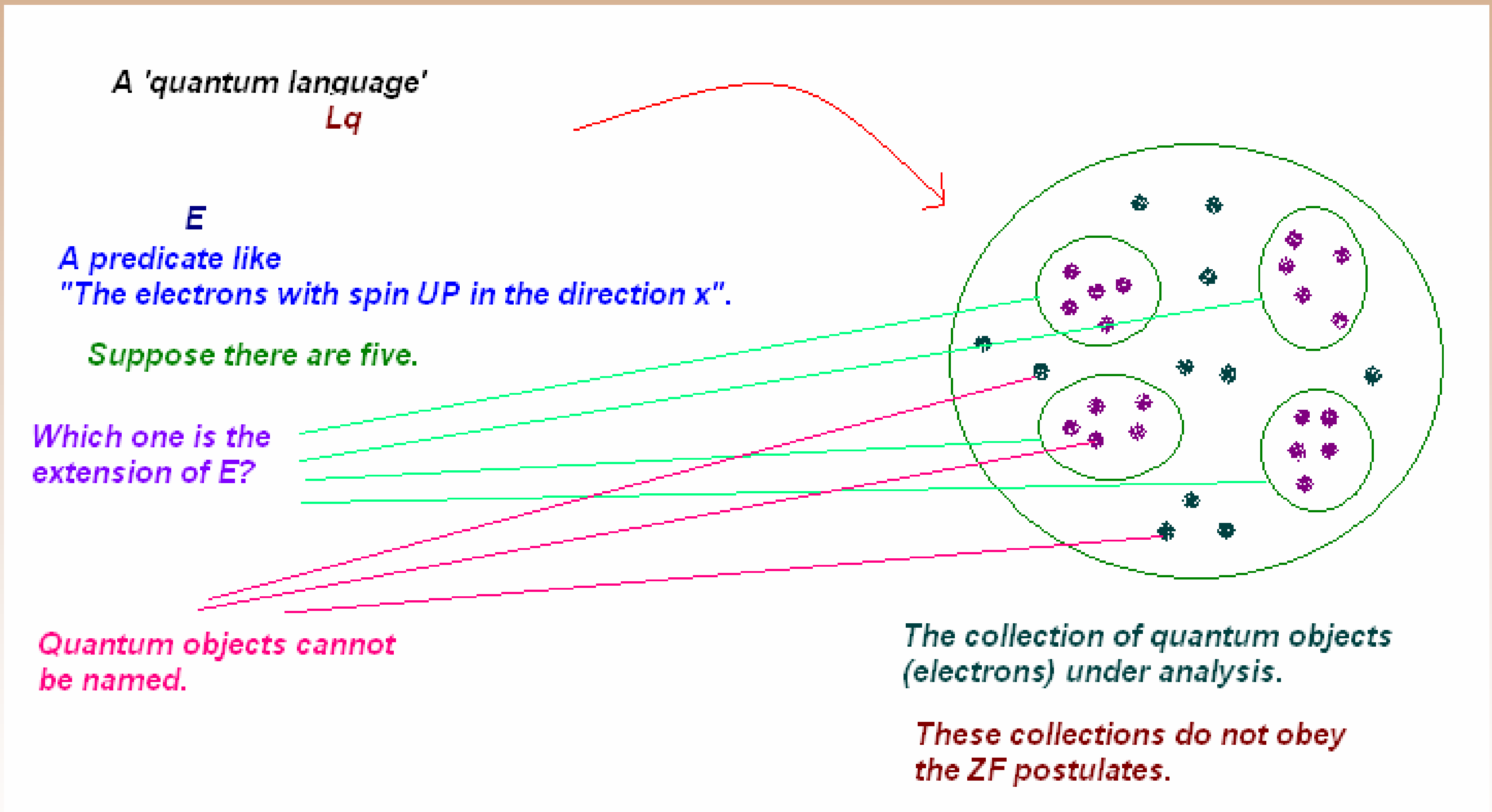
"We have all been taught to read 'Ex' as 'there exists an x such that', Since 'there is an x such that', considered as a phrase of English, has existential import, it is, perhaps, natural to suppose that 'Ex' does too." (S. Lavine, Synthese 2000)

The ‘standard’ procedure: the necessity to rule out the *object* from the formal counterpart of physical theories

(but of course not from the discourses of physics – read: its *pragmatic interpretation*).



‘Quantum semantics’ –how it should be

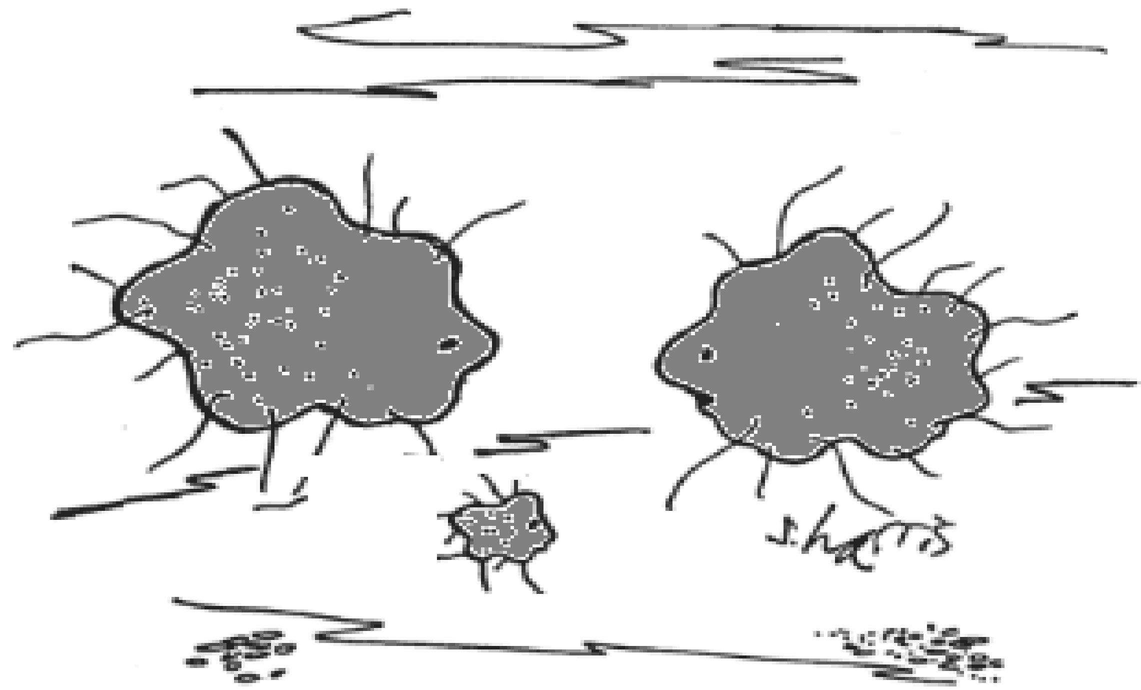


Which language?

- Heinz Post: Indistinguishability of quantum entities should be ascribed to them “right at the start”; they should be seem as *non-individuals.*
- There is no identity to lose.
- How to understand this *non-individuality*?
- Theories of Individuality:
  - Substance theories
  - Bundle theories
- Leibniz’ Principle of the Identity of Indiscernibles
  - $\forall F(Fx \leftrightarrow Fy) \rightarrow x = y$
- $I_a(x) =_D x = a$
- If *b* has all properties of *a*, then $I_a(b)$, thus $b = a$.

- The idea:
- The lack of ‘self-identity’: a language in which we cannot speak neither about identity nor about diversity of certain objects.
- Which kind of *objects*?

Indiscernible *objects*?



"He looks just like you.
But he looks just like me, too."

Methodological Principles

- There are quantum objects as described by quantum physics as *non-individuals*.
- They are classified in species or kinds
- How to intuitively think of them (if you have such needs): as elements of certain equivalence classes
- The properties and relations are invariant under permutations of indiscernible objects.
- The postulates of a certain QM hold (*e.g.,* Mackey's axioms)

The case of Quantum Physics

The case of classical particle physics (McKinsey, Suppes, Sugar 1953):

$$\text{CPM} = \langle P, T, s, m, f, g \rangle$$

P = the set of 'particles'

A right 'structure' to QP should not be extended to a rigid structure:

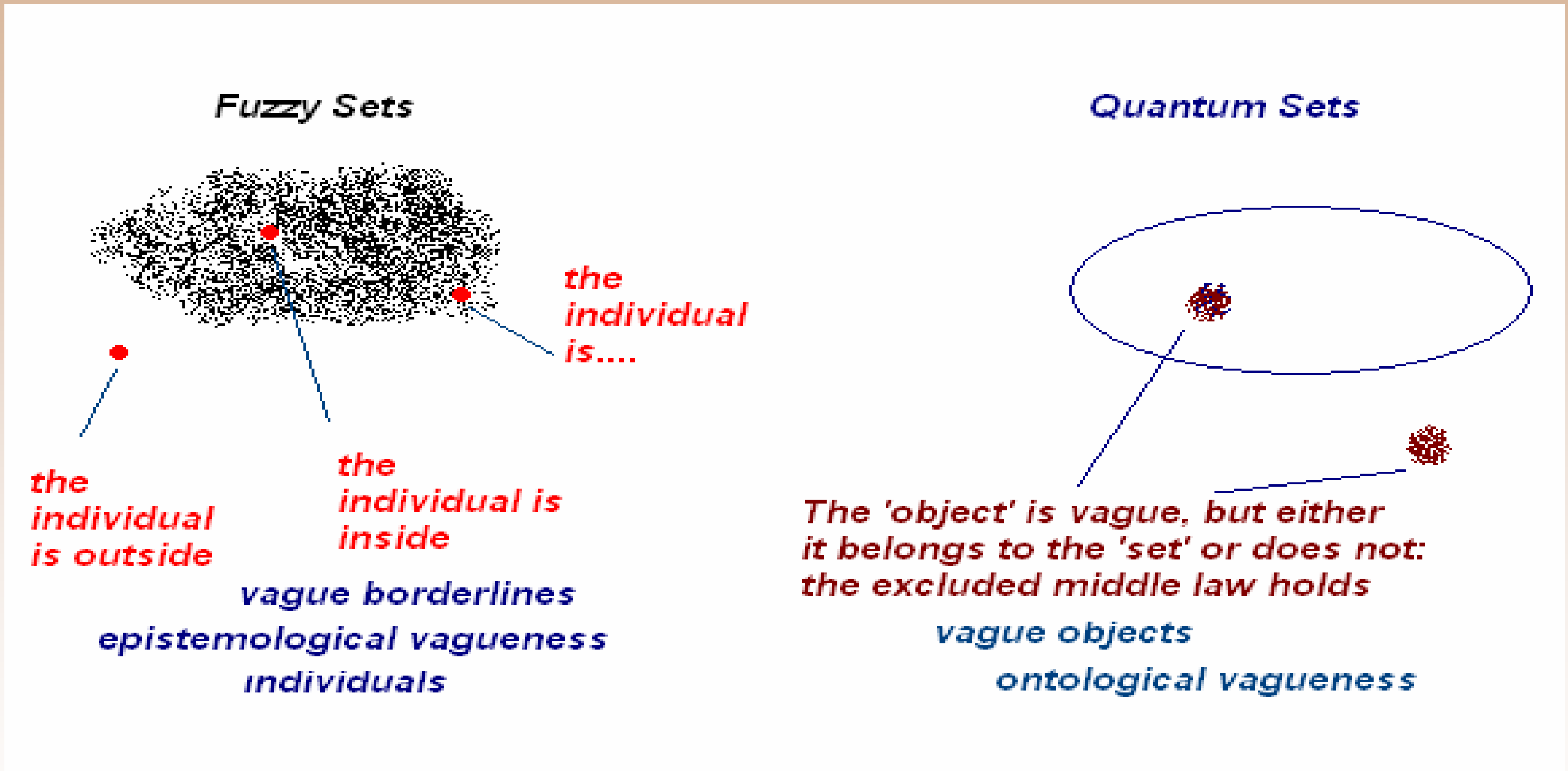
$$\text{QM} = \langle Q, \ldots., \ldots \rangle$$

Q = cannot be a *set*

Hence it cannot be performed within ZF.

(a new way to ask for a quantum logic?)

Why not Fuzzy Sets?



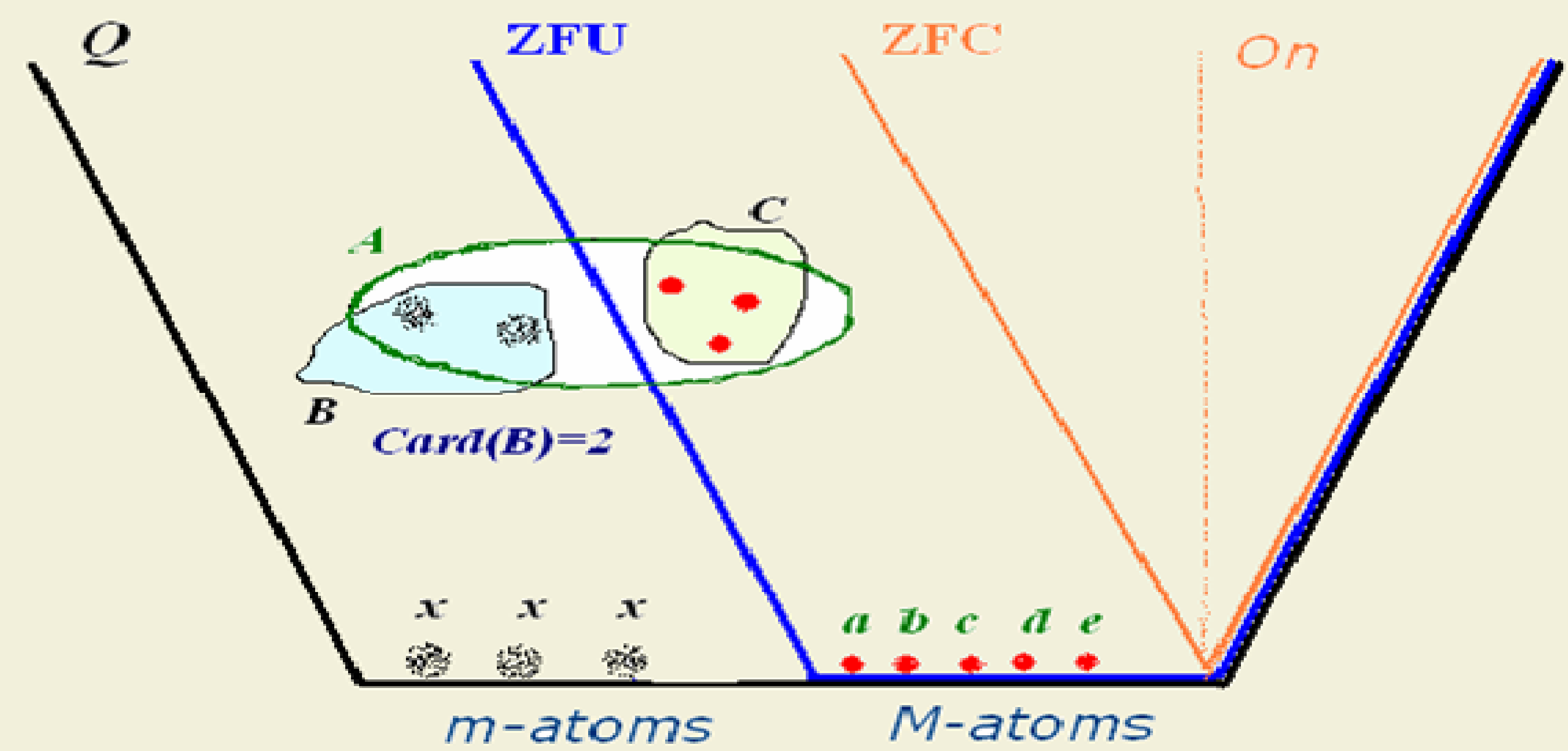
A logic with indiscernibility

- $\neg, \vee, \wedge, \rightarrow, \leftrightarrow, \forall, \exists$ logical symbols
  - m(x) microobject
  - M(x) macroobject
  - S(x) set
  - $x \in y$ membership
  - $x \equiv y$ indistinguishability
  - qc(x) the cardinal of x
- $Q(x) =_D \neg m(x) \wedge \neg M(x)$ quasi-set
- $x =_e y =_D (Q(x) \wedge Q(y) \wedge \forall z(z \in x \leftrightarrow z \in y)) \vee (M(x) \wedge M(y) \wedge \forall z(x \in z \leftrightarrow y \in z))$
  - extensional identity

- Standard postulates for classical 1st order logic
- $\equiv$ is reflexive, symmetric, and transitive
- $x =_e y \rightarrow (A(x) \rightarrow A(y))$, with the usual restrictions

- Theorems:
  - $x =_e y \rightarrow x \equiv y$ (but not conversely)
  - If x and y are both Sets or M-atoms, $x \equiv y \rightarrow x =_e y$
  - Hence $=_e$ becomes classical identity.
  - But for the m-atoms, only the relation $\equiv$ holds.
- Other ‘set’-postulates, similar to those of ZFU.

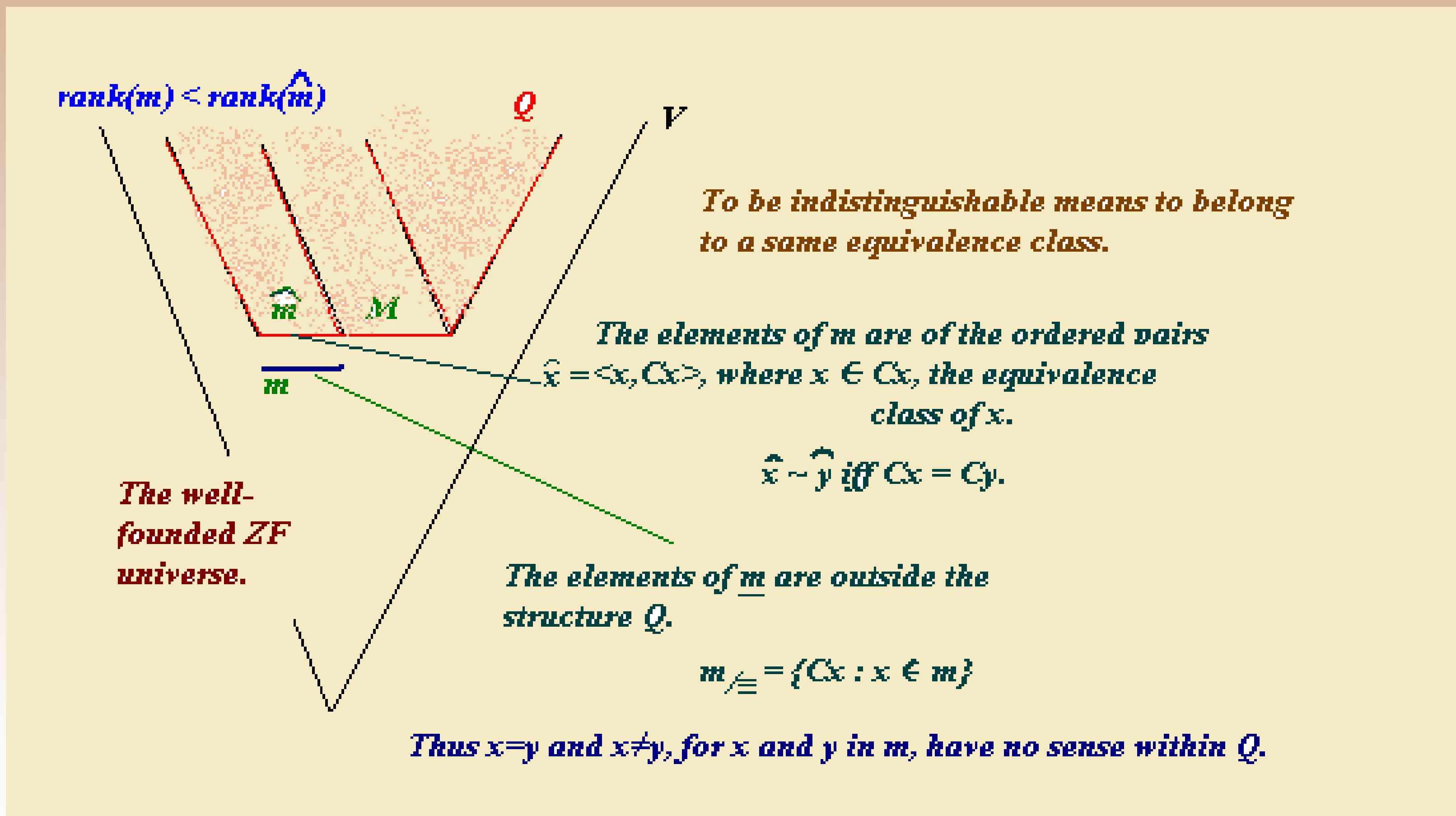
The Universe of Quasi-Sets



- The Quantifiers
  - “There exists x such that P”
  - Does the truth of this sentence compromises us with *individuals*?
  - (Quine’s ontological commitment)
  - The ‘meaning’ of quantifiers is ascribed by implicit definitions (the ‘operational’ definition of the quantifiers).
  - They are like a linguistic devices, acting as certain ‘operators’

- We shall be in trouble only if we are commited to *classical semantics*
- (where the extensions of the predicates are *sets* of individuals).
- The problem: to find an ‘pragmatic’ semantics which mirrors QP

The ‘standard’ semantics – in ZF



- A formal semantics (in the standard sense) for quantum theories cannot be provided within standard mathematics.
- We can discuss informally what is going on, but a formal account to this field can be provided only by a kind of *non standard semantics.*

- Quantum physics is an ineffable domain, one which we talk about those things of which nothing can be said (in standard languages).

- L. Wittgenstein (in the *Tractatus*)
    - "7. Whereof one cannot speak, thereof one must be silent."
        - Or to change language.
            - And logic !

Whither the particle grin?

- "Well! I've often seen a cat without a grin," thought Alice; "but a grin without a cat! It's the most curious thing I ever saw in all my life!"
- The particle grin (and the 'pragmatic' notion of *object*) remains...
- "(…) people reject quantum objects because they are different but all their argument shows is that there is nothing like classical objects in the quantum realm, not that there is no quantum object." (Auyang, S. Y., 1995)

http://www.webslingerz.com/jhoffman/chcat.html

Thank you